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Relationship between the strains in members and the end connections in an aluminum trussed space-frame tower

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RELATIONSHIP BETWEEN THE STRAINS IN MEMBERS

AND THE END CONNECTIONS IN AN ALUMINUM

TRUSSED SPACE-FRAME TOWER

by

William Carl Alsmeyer

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

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Approved:

In Charge of Major Work

Head of Major Department

Dean of Graduate College

Iowa State College

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TABLE OF CONTENTS

I INTRODUCTION

The objectives of this investigation are to study the
effect of end connections on the behavior of single-angle
compression and tension members in an aluminum space-frame
tower and, on the basis of the results of this study, to
develop practical design methods which are more rational
than those now used in structural design.

Many of the present day methods of designing structural
elements are necessarily arbitrary because of insufficient
experimental evidence and data concerning the behavior of
these elements in the actual structure. As this knowledge
becomes available more nearly rational methods of design
will be developed and used by the profession. The design
of single-angle truss members both in compression and in
tension, which this study covers, is one of the areas in
which this lack of knowledge is very acute and the design
methods specified in the standard structural design specifi-
cations are, therefore, necessarily arbitrary. Three gener-
ally used design specifications are those prepared by:

1. American Association of State Highway Officials, AASHO (1).

2. American Institute of Steel Construction, AISC (2).

3. American Railway Engineering Association, AREA (3).

To develop more rational methods of design than those presented in the foregoing specifications it is necessary to correlate the test data of members as integral parts of actual structures with laboratory test data of corresponding individual members and with previously or newly developed theory.

Three single-angle members of the tower were selected. Each of these three members was tested in tension and compression. Observations were first made on the members while they were in their normal positions in the tower; then each member was removed and observations were made in the testing machine under controlled conditions. In all of these tests detailed strain measurements were made. The strain measurements of the members in the tower were compared with those of the corresponding members tested individually in the machine. This comparison provided a reliable indication of the behavior of the members in the tower on the basis of the controlled conditions in the testing machine. Then, after the behavior of the members had been established, a proposed theory was developed. In order to provide the designer with a readily applicable design procedure, the proposed theory was modified.

Owing to the limited time and funds available one type
of single-angle member in a trussed space-frame tower struc-
ture was used. Therefore, the results are strictly applicable
to this type of member and structure only. A general applica-
tion of the proposed design methods should not be undertaken
without additional experimental work on other types of members
and trussed structures.

No evidence was found in any of the available literature
of studies which were correlated similarly to the studies
undertaken in this investigation. Many examples of tests of
individual members in testing machines under controlled con-
ditions are available. Most of these tests have been ultimate
tests with little detailed strain data recorded. Also, the
majority of these tests have been compression tests of large
rolled or built-up sections such as are used in bridge trusses
and building frames. There have been a considerable number of
projects where strain measurements were taken on actual struc-
tures, but most of these structures have been bridges or
buildings. There is also available a great deal of theory on
the analysis and design of trussed structures and their ele-
ments.

In many cases theoretical investigations have been carried
out in conjunction with either tests of individual elements
or tests of a structure, but not with both for the same
structure.

A few references are cited here which have a direct bearing on this problem. Batho (4) conducted tests on single-and double-angle tension members to determine the effect of end connections on stress distribution. The main objective of his study was to determine the effect of lock or lug angles at the ends of the members. Equal leg angles were riveted to two end gusset plates which were attached firmly in the testing machine. Longitudinal strain measurements were taken with mirror extensometers at sections near the center and ends of the members. Strain distribution curves for each of these sections are given, (4, p. 145), both with and without lug angles. For the center sections of the single-angle members without lug angles the strain across the attached leg is nearly constant. The maximum difference in strain between the two edges of this leg is not more than five per cent. Two of his conclusions are: (4, p. 172)

a) That the assumption of a planar distribution of stress is justified in such members as are considered here, except perhaps close to the end connections, and that ordinary theory may therefore be applied to an analysis of the distribution of stress in these members.

b) That in single-and double-angle members connected at their ends by means of rivets to wide and rigidly held gusset plates the stiffness of the gusset plate in its own plane has a considerable effect on the distribution of stress in the member, there being in every case a particular stiffness which will give the least maximum stress in the member for a given load.

Templin, Hartmann and Hill (5) conducted tests on a pair of 28-foot aluminum alloy trusses which were spaced 42 inches center to center and fastened together by lateral bracing in the planes of the chords and by transverse cross bracing between the posts. The truss members were all equal leg double-angle members, but arranged so that all of the web members in one truss could be changed to single-angles by removing the outside angles.

Detailed strains, deflections and changes in lengths of members were measured. The observed data were utilized for several purposes. Observed primary stresses (axial loads only), secondary stresses (all other stresses) and deflections were compared with computed values. Good agreement was found except for the secondary stresses where the agreement was only fair.

In the section of the report devoted to the behavior of single-angle members, the following statement is made: (5, p.19)

> There are too many unknown factors affecting the behavior of the single-angle members of the truss to permit a complete and accurate analysis of their action. The direct force in a member is known from the average of the measured stresses, but the manner in which this direct force is applied and the magnitude and direction of the bending and resisting moments acting at the ends of the member are very indefinite. The direct load is applied to the member in a series of unknown increments through the rivets of the joint. Unknown bending moments at the ends of the member are introduced by the eccentricities of the connections, the rigidity of the joint and the deflection of other members

meeting at the joint. The effective length of the
member is also indefinite because of the variable
moment of inertia of the portions of the member
included in the joints.

The logical method to employ in calculating the
maximum stress in a single angle compression mem-
ber would be to consider bending of the member
about each of the principal axes separately and
then to combine the results. The difficulty
lies in the fact that the bending moments at the
ends of the member cannot be evaluated. It is
not always safe to consider the member as an
eccentrically loaded, pin-ended column having a
length equal to the distance between panel points,
which deflects only in a direction normal to the
plane of the gusset plate. In the case of member
U_4L_4 the stress so computed, using a load equal
to the average measured stress times the area,
was only about half as great as the maximum stress
measured when the cross-frames were unbolted. In
this computation the secant formula was used and
the eccentricity was assumed equal to the distance
from the back of the angle to the centroid of the
section.

The member U_4L_4 cited is a post member directly over
the point of loading and its increase in maximum unit stress
when the cross-frames were removed was more than 100 per
cent. The maximum unit stress in the adjacent diagonal mem-
ber, however, was increased only ten per cent. The state-
ment is made: (5, p. 19) "These increases are consistant with
what would be expected because the cross-frames, when con-
nected, give considerable support to the posts but very little
support to the diagonals."

Diagrams are plotted (5, p 37) for each of the single-
angle web members showing the longitudinal stress distribu-

tions on five cross-sections of each member. For each member the stress on the attached leg of the angle is constant at some section near the center of the member. The stress is a maximum at the free edge of this leg at one end of the member and on the supported edge at the other end of the member.

II EXPERIMENTAL PROCEDURE

A. Description of Tower

For this investigation an available aluminum trussed
space-frame tower, which had been designed and constructed
by C. W. Cunningham[1] for another project, was used. A photo-
graph and a line drawing of this tower are shown in Figures
1 and 2 respectively.

All of the members in this tower are single-angle mem-
bers. Each of the four corner posts is a continuous member
throughout its length while each of the diagonal and ring
members is an individual piece connected to the posts by
gusset plates and bolts as shown in Figures 3, 4 and 5.
The post members are 1 3/4 x 1 3/4 x 3/16 inch angles, the
ring members are 1 3/4 x 1 3/4 x 1/8 inch angles and the
diagonal members are 1 1/2 x 3/4 x 1/8 inch angles. The
post members are connected by both legs to the gusset plates
while the ring and diagonal members are connected by one leg
only. For the diagonals the longer legs are attached. The
legs of the post and ring members are spread so that each
leg of each angle lies in one of the planes of the tower.

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[1]Professor Charles W. Cunningham, Department of Civil
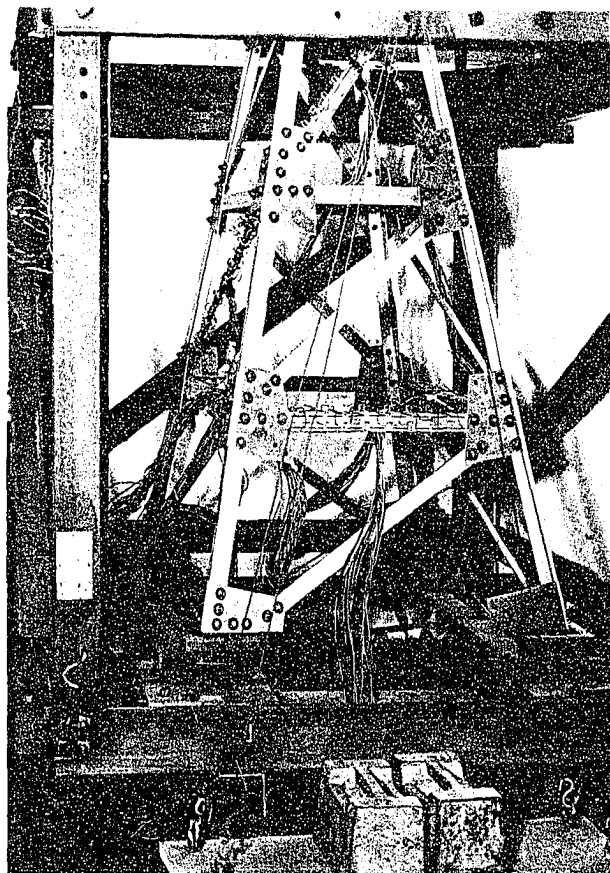Engineering, The College of the City of New York.
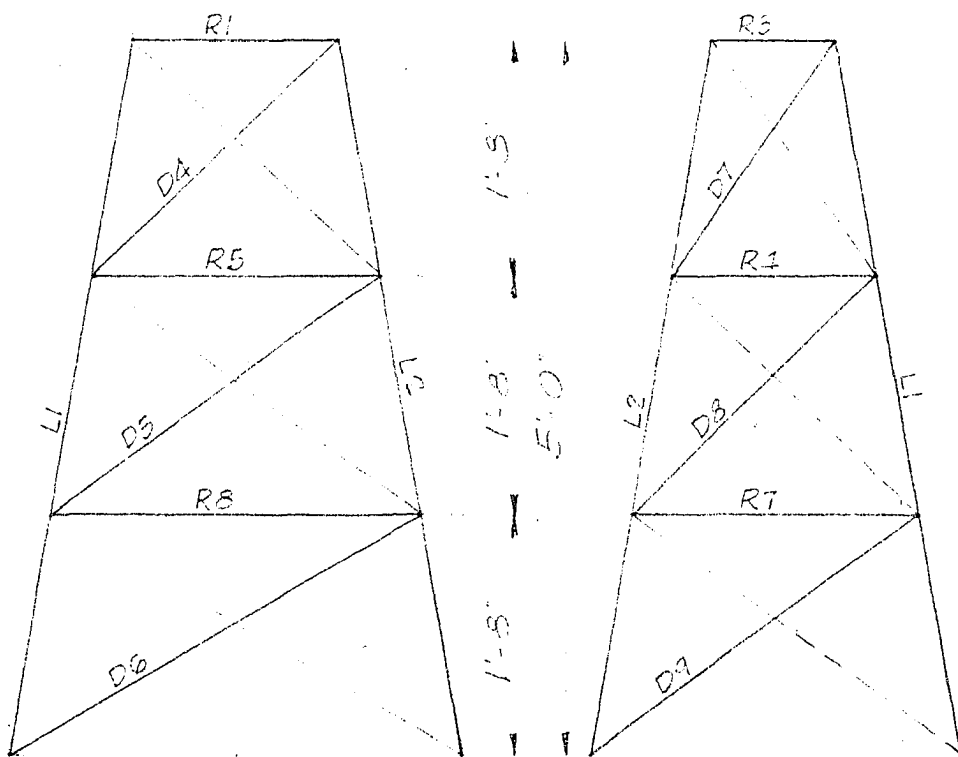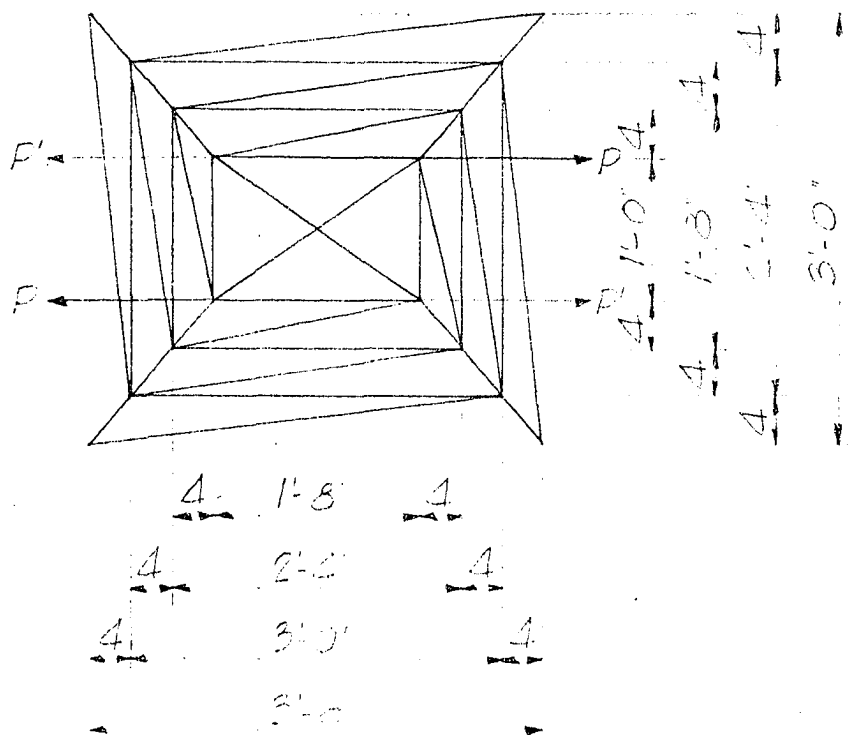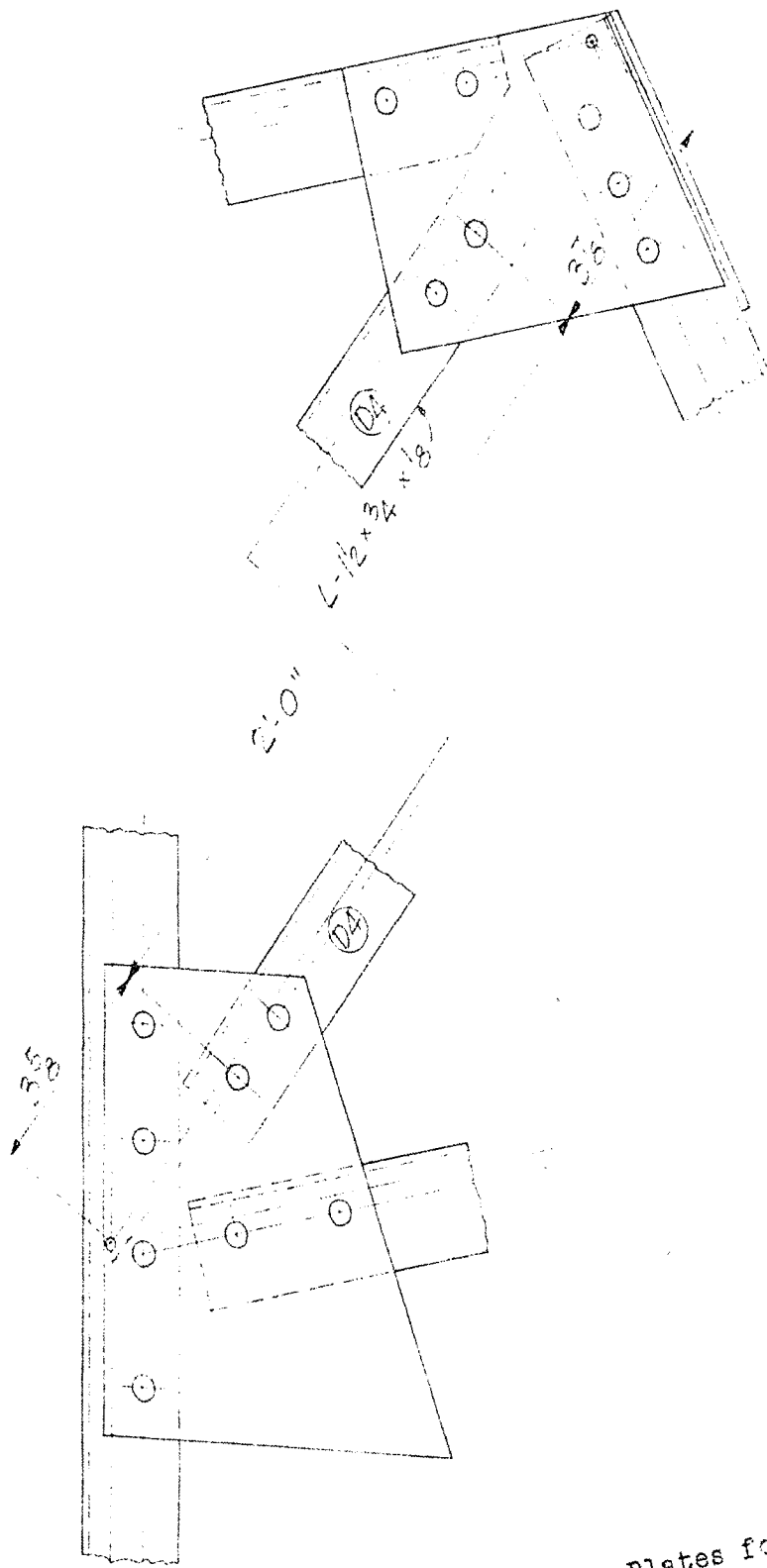
Figure 1. Photograph of Tower

Figure 2. Line Drawing of Tower

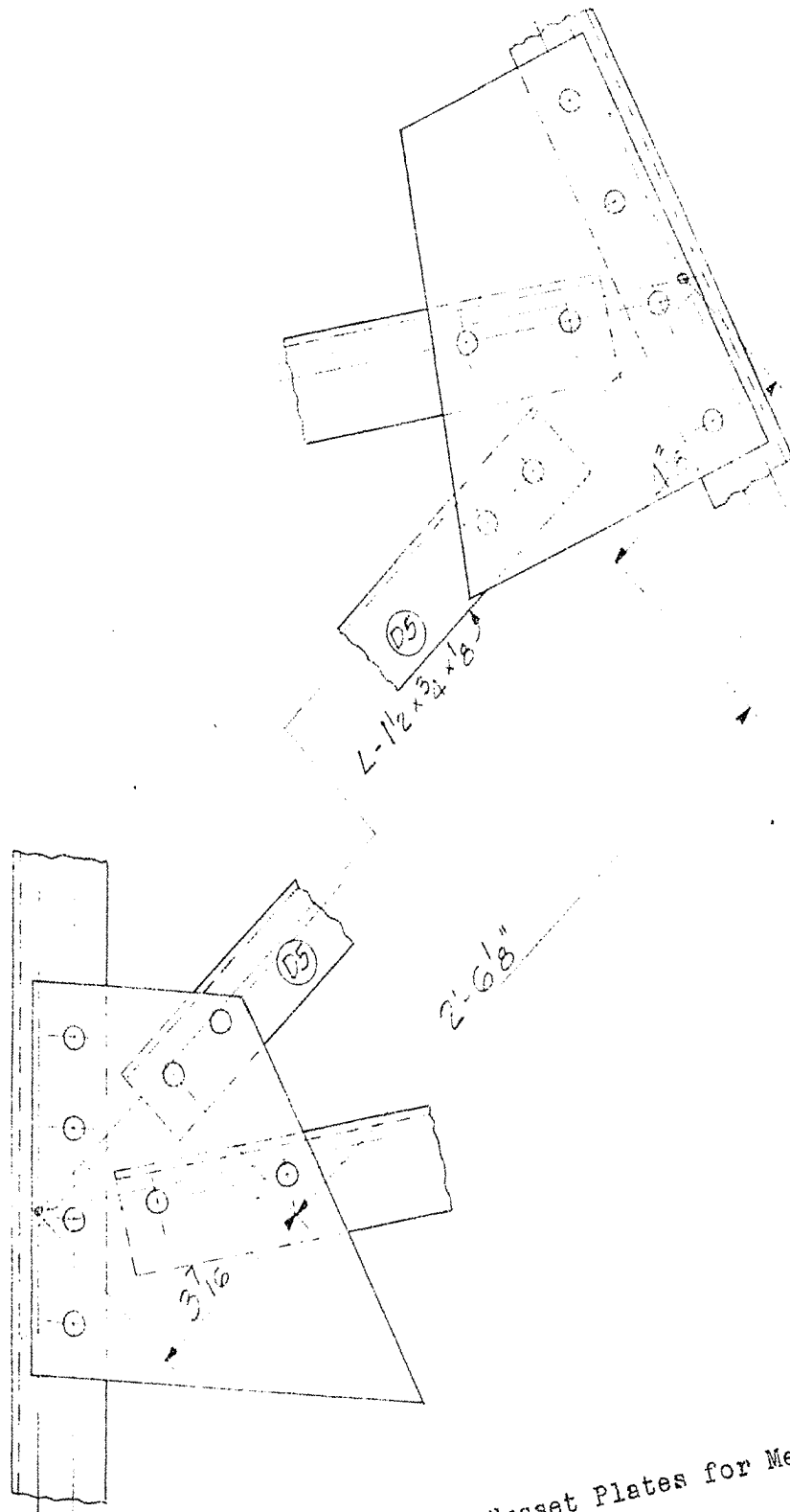Figure 3. Details of Tower Gusset Plates for Member D4.

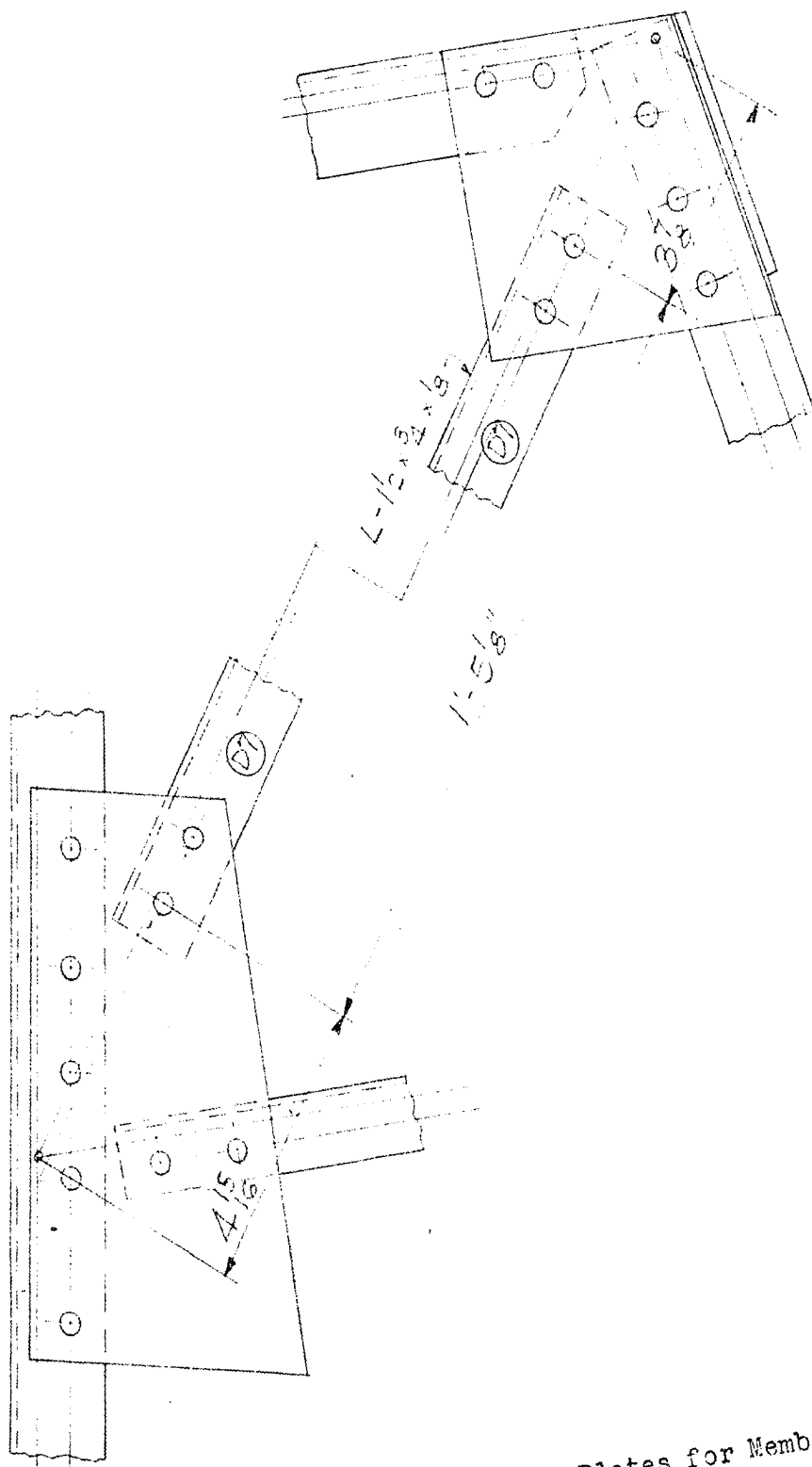Figure 4. Details of Tower Gusset Plates for Member D5.

13

Figure 5. Details of Tower Gusset Plates for Member D7.

The connections in the tower are made with 1/8-inch 24-ST aluminum alloy gusset plates and 3/8-inch steel bolts, except that 3/16-inch gussets are used at the four top joints.

The three diagonal members D4, D5 and D7 were chosen for this investigation as typical of one type of single-angle truss member and duplicate members were fabricated for testing purposes. The locations of these three members in the tower are shown in Figure 2; the details of each member and its connections are shown in Figures 3, 4, 5 and 6. All of the original members are of 17-ST aluminum alloy and the new diagonal test members are of 14-SW aluminum alloy, the elastic properties of which are considered to be equivalent by the manufacturer.

The original loading system, which consists of a loading frame and several loading mechanisms, was used with slight modifications. This system is designed so that the tower is allowed to assume any natural displacement without affecting the magnitude and direction of the applied loads. The loading frame is constructed of structural steel angles and channels. The loading mechanism, a line drawing of which is shown in Figure 7, consists of a load platform and a 20 to 1 ratio lever system. Two of these mechanisms were placed on the loading frame so as to apply the horizontal torque loads P or P', as shown in Figure 2. Each of these pairs of loads applies a torque in the top plane of the tower.

MEMBER D4

MEMBER D5

FIG. 6. DETAILS OF TEST MEMBERS AND

			47
			43
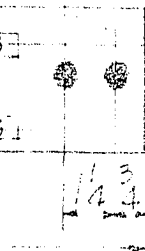
17	19	7	A	23	35	39	43	47
2	14	8	12	34	37	41	44	48

| | 19 | 17 | 19 | 25 | 37 | 41 | 45 | | |
| 18 | 20 | 21 | 24 | 38 | 42 | 46 | | | |

35	37	45	47
36	43	44	48

37	41	45
38	46	46

$L-1\frac{1}{2}\times\frac{3}{4}\times\frac{1}{8}$
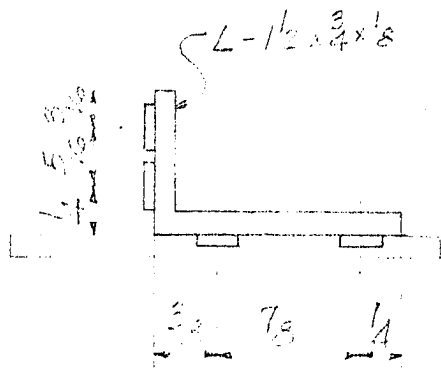
TYP SECTION

MEMBERS AND LOCATION OF GAGES

Figure 7. Line Drawing of Loading Mechanism

B. Description of End Fixtures

A pair of end fixtures was designed and constructed for
the testing of the three diagonal test members both in ten-
sion and in compression in a 20,000 lb. hand operated uni-
versal Tinius Olsen testing machine. A drawing showing
details and assemblies of these fixtures appears in Figure
B 1, Appendix B. Three end conditions can be obtained by
different combinations of the elements of the fixtures.
These are:

1. Rigidly fixed, Figure 8c.

2. Hinged in any one direction, Figure 8b.

3. Hinged in two directions normal to each other,
    Figure 8a.

The hinges in these fixtures are produced by ball bearings
in races as shown in Figure B1, in Appendix B.

C. Description of Strain Measuring Equipment

The strains in the members were measured with SR-4 bond-
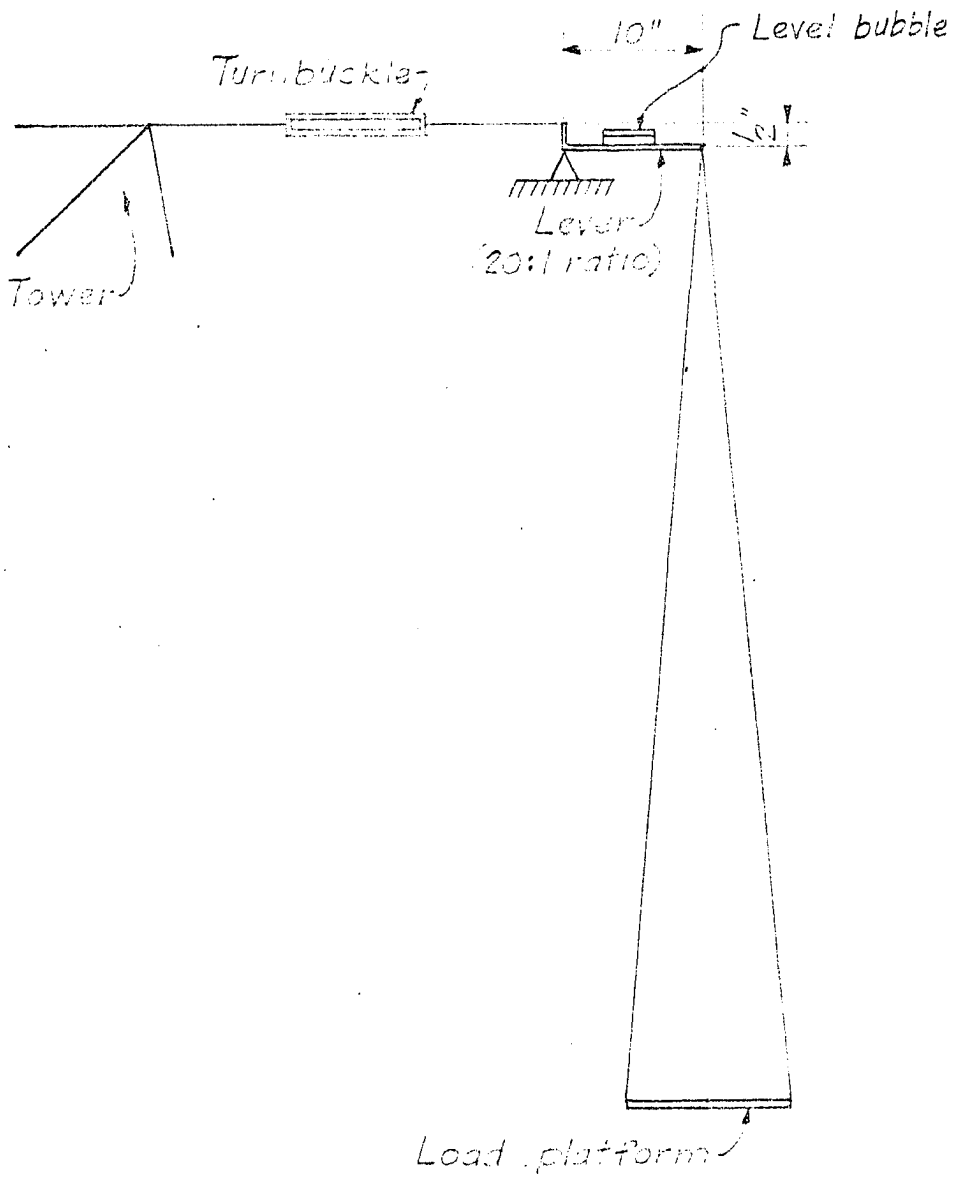ed electric resistance strain gages Type A-12 as manufactured
by the Baldwin-Lima-Hamilton Corporation.[1] Since these gages
are now well known and very widely used no detailed descrip-
tion of the gages and their theory of operation will be

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[1]Formerly Baldwin Locomotive Works.
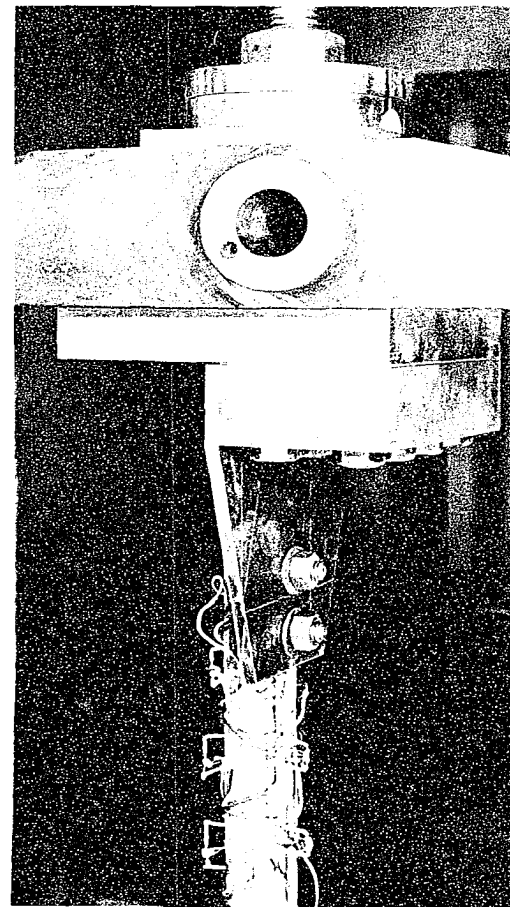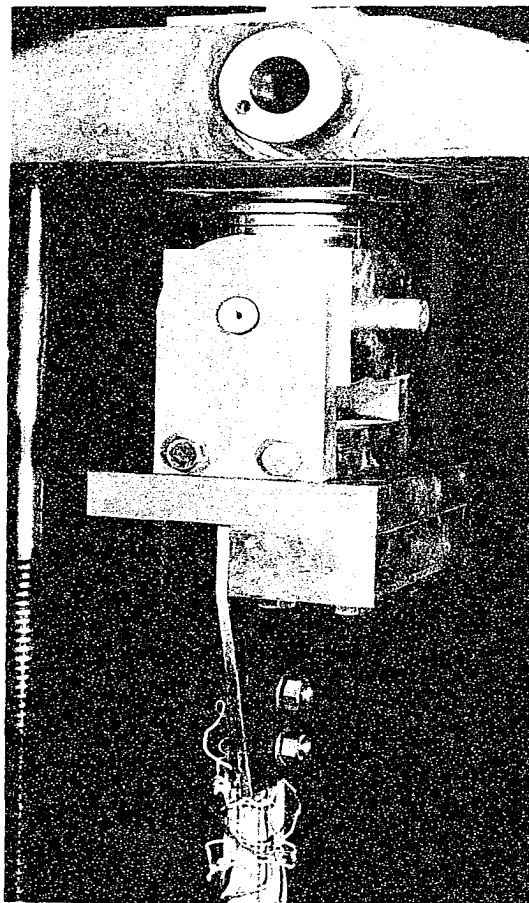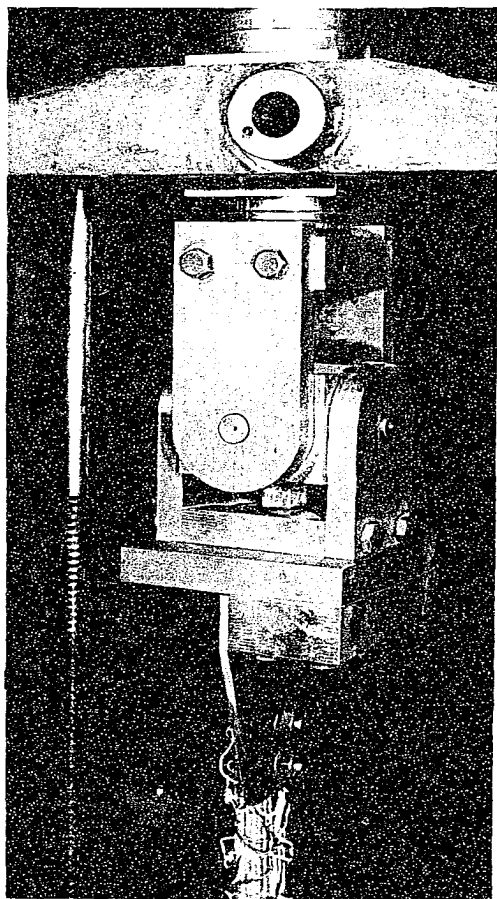
a. Hinged Two Directions   b. Hinged One Direction   c. Rigid-Fixed

Figure 8. Photographs of End Fixtures

presented here. References (6), (7), (8) and (9) are a few

of the available sources of information on the SR-4 strain

gages. The gages were bonded to the test members with SR-4

Cement in the manner prescribed by the manufacturer and are

located on the members as shown in Figure 6.

The strains at the locations of the gages were measured

and recorded on circular charts with a 48-channel Switching

Unit and an SR-4 Strain Recorder. Both of these units are

sold by the Baldwin-Lima-Hamilton Corporation and when used

together are referred to as SR-4 Scanning and Recording

Equipment. A description of this equipment and its opera-

tion may be found in (10). The following statement (10, p.4)

is made concerning the overall accuracy of this equipment:

> When used with SR-4 Type A-1 gages properly applied
> to the test structure by a technique which is out-
> lined with each package of these gages, the overall
> accuracy of the SR-4 Scanning Recording Equipment
> may confidently be expected to be within one per
> cent of full scale range in either of the three
> normal ranges.
>
> Expressed in absolute values (instead of percent-
> ages) general experience indicates a better ac-
> curacy at low strain values than would be indicat-
> ed by percentage values. To express these values
> quantitatively, however, becomes practically im-
> possible because of variables not under our
> control.

The Scanning and Recording equipment has three normal ranges

of 0 to 2000, 0 to 5000 and 0 to 10,000 micro-inches per

inch. In this investigation the 0 to 2000 range was used

for all tests.

Nedderman (11, p. 137) compared the SR-4 Scanning Recorder with the SR-4 Type L Strain Indicator and the Huggenberger Tensometer. His results indicate that for strain readings of 120 micro-inches per inch and above the range of variation among the three instruments does not exceed plus or minus three percent and for readings of 250 micro-inches per inch and above the variation is less than plus or minus two percent. However, for lower values such as 50 or 60 micro-inches per inch the percentage of variation was plus or minus seventeen percent.

D. Tests of Members in Tower.

The three diagonal test members were placed in their normal positions in the tower and the loads P were applied to the tower by the loading mechanisms. The torque, due to the loads P in the top horizontal plane of the tower, produced tension loads in each of the diagonal members. By shifting the positions of the loading mechanisms the loads P' were applied to the top plane of the tower. The reverse torque produced by these loads applied compression loads in each of the diagonal members. Thus, each of the three test members was tested in both tension and compression. Since the Scanning Recorder could accomodate only 48 gages, and since each of the test members had from 32 to

48 gages it was necessary to test one member at a time. For each member both in tension and compression the loads were applied to the tower by placing equal weights simultaneously on the two loading platforms in increments of 10 lb. to a maximum of 100 lb. These weights, because of the 20 to 1 ratio lever system, applied P or P' loads to the tower in increments of 200 lb. to a maximum of 2000 lb. The levers were kept in their proper positions by means of attached level bubbles and adjustment of the turnbuckles so that the 20 to 1 ratio would remain constant. The maximum loads applied to the tower were limited by the yield strength and the difficulty of adjusting the turnbuckles. Each of the three members was loaded three times as outlined above and for each increment of load strain readings were recorded for each strain gage with the Scanning Recorder. An examination of the circular charts, on which the strains were recorded, showed a regular spacing of lines indicating smooth data. Because of this regularity of spacing only a few of the values were used to plot the necessary curves.

The load on a test member in the tower produced by a load applied to the tower was determined as follows: It was assumed that the strain distribution on each cross-section of the member was planar and since all of the recorded strains indicate unit stresses below the elastic limit

for the material this is a reasonable assumption. On the basis of this assumption the strain readings for the four gages numbered 23, 24, 25 and 26 at the center cross-section of the member were projected over the area of the member and integrated to determine the strain volume. The load on the member was then determined by using the modulus of elasticity, E, for the material in the members as determined in Appendix C. The same procedure was followed using the strain readings for each of the two cross-sections adjacent to the center of the member. The results of these three calculations were then averaged as the load on the member in the tower. The loads thus obtained for members in the tower are comparable to the loads applied to the same members in the testing machine.

E. Tests of Members in Testing Machine.

Each of the three tower test members was tested in the testing machine both in tension and compression with four different controlled end conditions imposed by the end fixtures. Typical tension and compression set-ups in the Tinius Olsen 20,000 lb. hand operated universal testing machine are shown in Figure 9 and typical end fixture arrangements are shown in Figure 8. The four controlled end conditions used were:

23
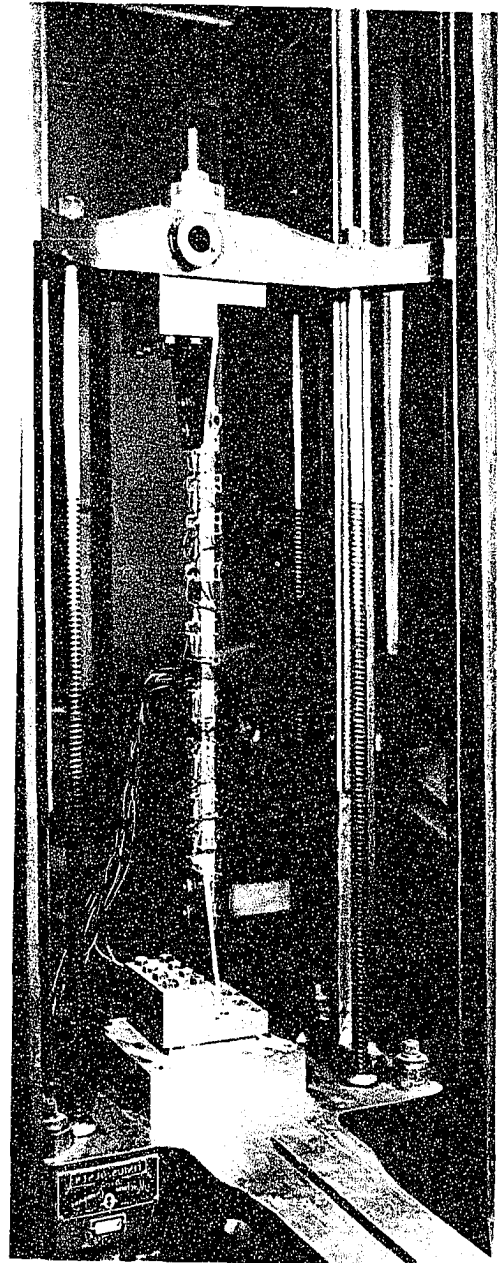
a. Tension   b. Compression

Figure 9. Typical Test Set-ups in Testing Machine

1. Rigidly fixed.

2. Hinged parallel to the gusset plates.

3. Hinged normal to the gusset plates.

4. Hinged both parallel and normal to the gusset plates.

For each of the four end conditions, a complete series of tests for each of the three test members was run using two 1/3-inch 24-ST aluminum alloy gusset plates as shown in Figure B1, Appendix B. For member D4 only, two additional series of tests were run using two 1/3-inch and two 1/4 inch steel gusset plates which are the same shape as the aluminum gussets. The designation for each of the test runs is given in Table 1.

In all of these tension tests the loads were applied in increments of 200 lb. to a maximum of 2200 lb. In the compression tests the load increments and the maximum loads varied with the individual members, the end conditions and the types of gusset plates. The magnitude of each of these maximum loads was dependant on either the critical load or the magnitude of the strains recorded. For each member, end condition and gusset plate type, both in tension and compression a test run was made with the outstanding leg of the angle at each of the four cardinal directions of the testing machine. In the early tests the members were also reversed end-for-end for a total of eight tests per end

condition, but an examination of the results indicated that this was not necessary, and on subsequent tests the members were not turned end-for-end. For each test strain readings were recorded at each increment of load for each strain gage with the Scanning Recorder. The four strain readings for each gage were averaged and these are the values which are used. Again, because of the regularity of spacing of lines on the circular charts, only a few of the values were used to plot the necessary curves for the tension tests.

For the compression members, however, it was necessary to use all of the values to plot the necessary curves.

III PRESENTATION AND DISCUSSION OF RESULTS

In all of the experimental work in this investigation all of the strains or stresses in the test members were within the elastic range of the material used. Thus, the results obtained are equally applicable to any other structural material within its elastic range.

The sign convention for stresses and strains is plus for tension and minus for compression.

A. Behavior of Members in Tower.

The specific behavior of a member, which is an element of a complete structure, is usually influenced by so many factors that the interpretation of the exact action does not lend itself to a rigorous mathematical analysis. However, as the influence of these factors becomes clarified, theory based on sounder assumptions can be developed for the design of such a member.

For the single-angle members in this trussed structure three of the more important basic factors considered are secondary bending, end conditions and elastic curvature. All three of these factors are directly affected by the type and efficiency of the end-connections, and the general framing of the structure. Irregularities resulting from rolling and fabricating these members are not considered because they appeared to be negligible.
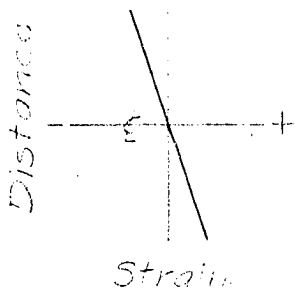
1. Primary strains.

In the tower the longitudinal loads which are applied to each end of a test member through the gusset plates are eccentric to the axis of the angle. The eccentricities at the ends of a member would be practically equal both in magnitude and direction. The differences would be caused by the various shapes and connections of the gusset plates at the two ends of the member as shown in Figures 3, 4 and 5. Each of these eccentric loads can be resolved into an axial load at the centroid, and a bending moment about an axis through the centroid of the angle. The bending moments at the two ends would be approximately equal, but opposite in direction.

The longitudinal strains, for a particular gage line along the length of a member, produced by the axial loads would be constant from end to end of the member. Those resulting from the end moments would be practically constant from end to end. In addition there would be strains resulting from the elastic curvature of the axis of the member caused by the eccentricities. These strains would vary from zero at each end to a maximum near the center of the member. The combination of these three strain components will be referred to as primary strains. A curve representing these primary longitudinal strains for a
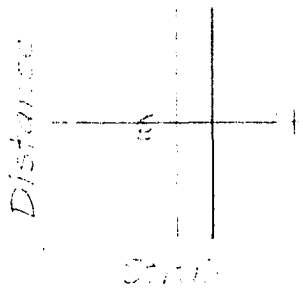
particular gage line would be approximately symmetrical about
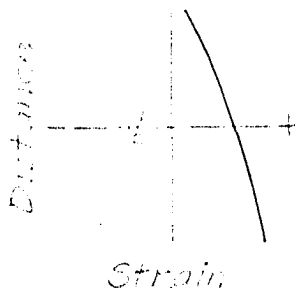the center-line of the member, as illustrated in Figure 10a.
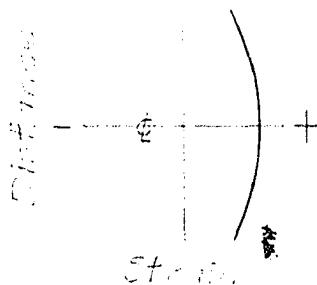
29
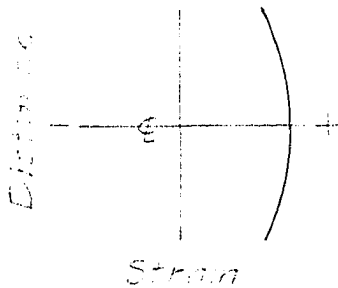
Figure 10. Typical Longitudinal Primary and Secondary
Strain Curves.

2. Secondary strains.

When the tower distorts as loads are applied the re-sistance to rotation of the joints will produce additional bending in a member as a result of the relative displacement and rotation of its ends. The resulting bending moments at the two ends may be either in the same direction or in opposite directions. This bending and the resulting strains will be referred to as secondary bending and secondary strains.

Typical longitudinal secondary strain curves for one gage line for each of the two possible conditions are shown in Figure 10b and 10c. When the secondary end moments are in the same direction the combined strain curve for primary and secondary strains is as shown in Figure 10d. The effect of this type of secondary bending is to rotate the primary strain curve as shown in Figure 10d. When the secondary end moments are in the opposite direction the combined strain curve for primary and secondary strains is as shown in Figure 10e. The effect of this type of secondary bending is to translate the primary curve as shown in Figure 10e.

Thus, it is possible to identify the type of secondary bending in a tower member by the orientation of its strain curves for various gage lines along the member.

The longitudinal strain curves for gage lines I, II, III
and IV for the tower test member D4 in compression are shown
in Figures 11 and 12. The curves for gage I appear to be
symmetrical but if the plotted points at each end are ignored
the remainder of each of these curves is rotated slightly.
Gage line I is on the outer edge of the projecting leg of
the angle and at the end of the member the strain readings
would be very unreliable because of the transfer of load
from the gusset plate to the angle. The curves for gage II
are slightly rotated, while those for III and IV are rotated
considerably. The rotation of all four of these curves in-
dicates the presence in the tower data for member D4 in
compression of secondary bending produced by end moments in
the same direction. Similar curves for D4 in tension, and
D5 and D7 in compression and tension appear in Figures A1
to A10 inclusive in Appendix A. All of these curves are
rotated similar to those for D4 in compression. Therefore,
these are secondary strains of the type produced by end
moments in the same direction in all of the tower test data.
These secondary strains are zero near the center of the mem-
ber and maximum at the ends. Consideration of these secon-
dary strains must be given whenever the strain data for the
tower tests are used.

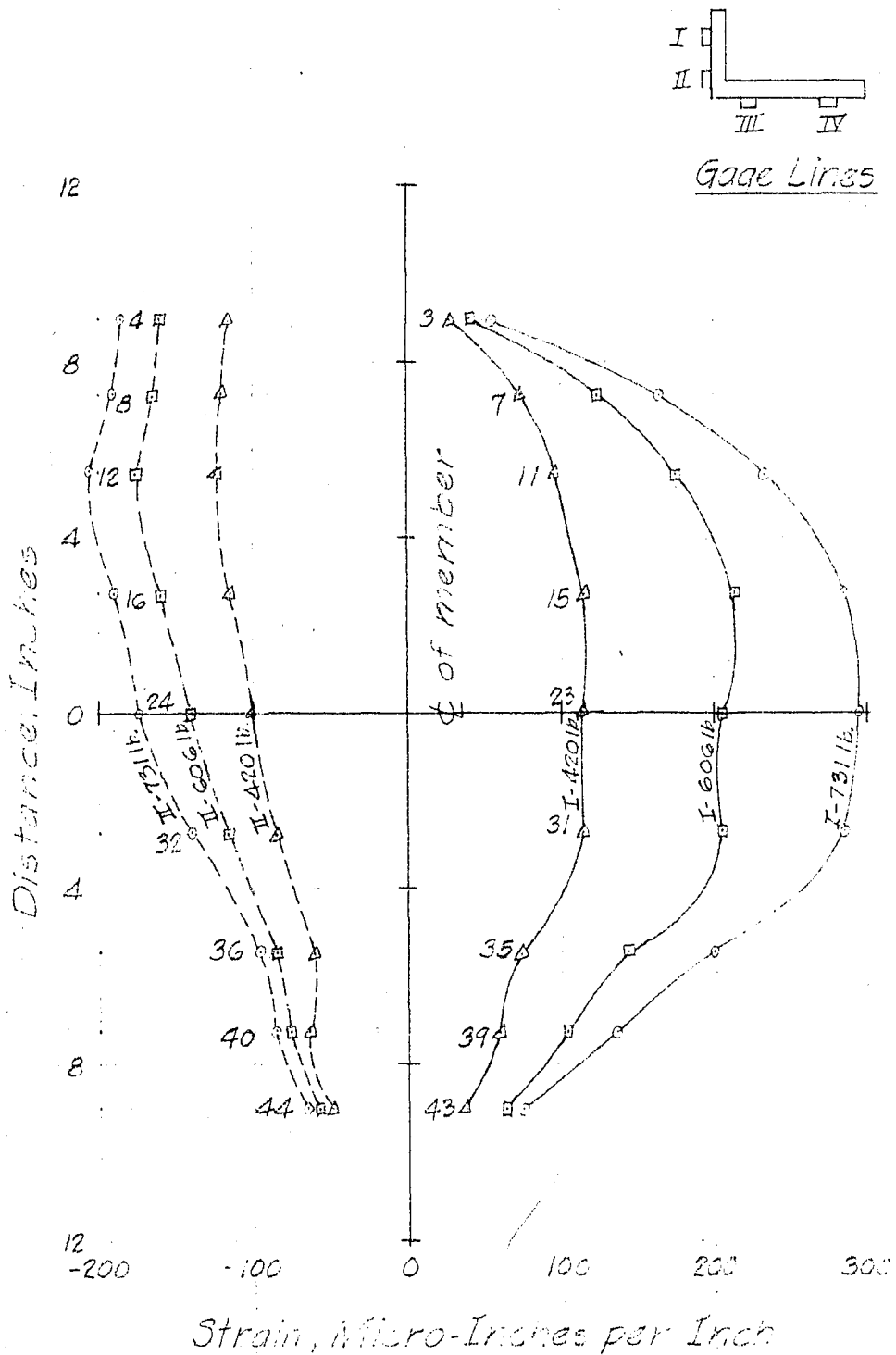Templin, Hartmann and Hill (5, p.37) had the same type

32

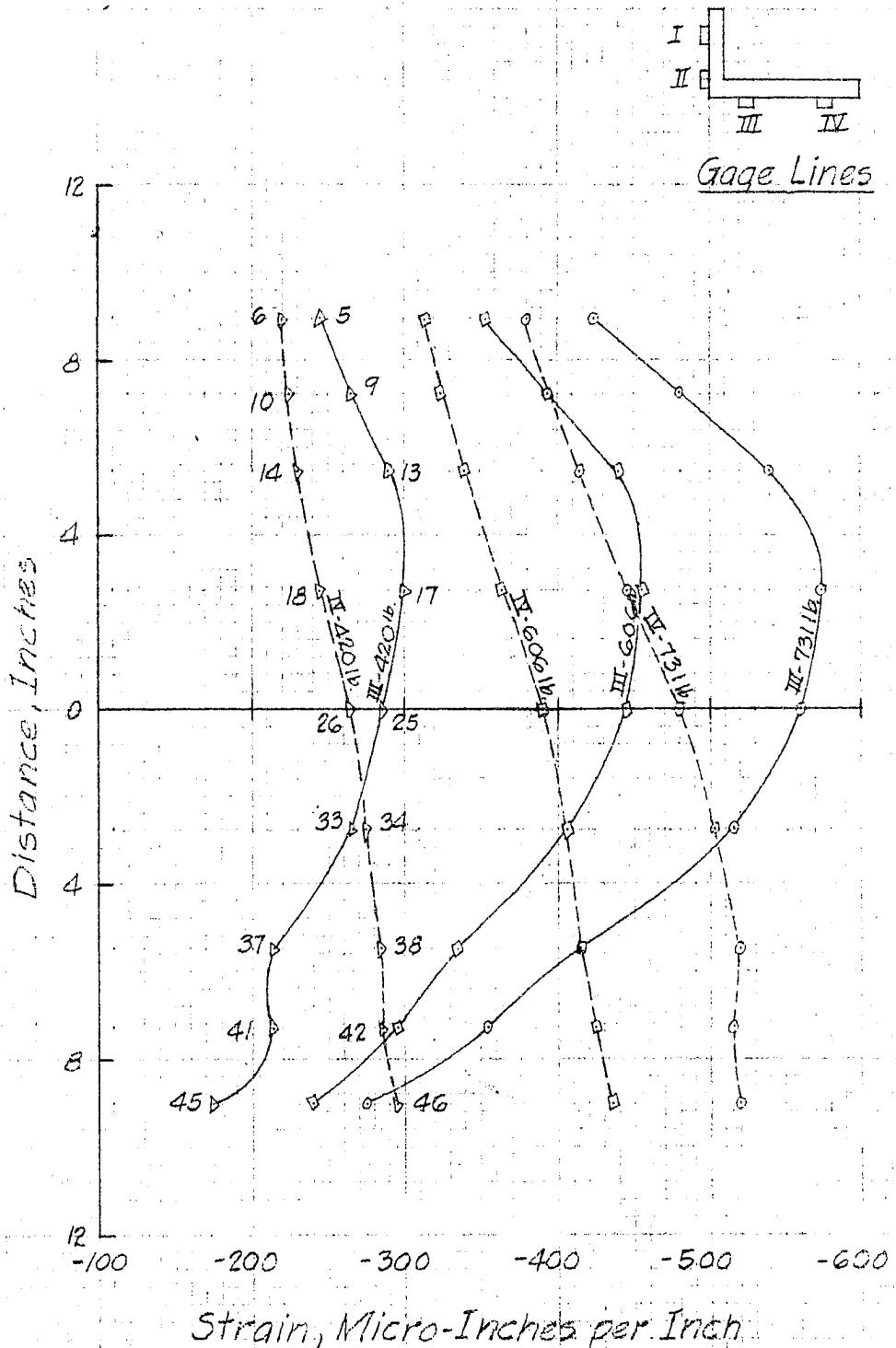Figure 11. Member D4-CT, Longitudinal Strain Curves for Gage Lines I and II.

Figure 12. Member D4-CT, Longitudinal Strain Curves for
Gage Lines III and IV.

34

of stress or strain distribution in all of their single-angle
web members as occurred in the tower indicating the presence
of the same type of secondary bending in their trusses as in
the tower.

In the design of truss members the additional stresses
produced by the secondary bending are usually not considered
unless they increase the critical primary stresses a stipu-
lated amount. The AASHO specification (1, p.167) includes
the following statement concerning secondary stresses:

> The design and details shall be such that secondary
> stresses will be as small as practicable. Secon-
> dary stresses due to truss distortion or floor-
> beam deflection usually need not be considered in
> any member the width of which, measured parallel to
> the plane of distortion, is less than one-tenth of
> its length. If this secondary stress exceeds 4000
> pounds per square inch for tension members and
> 3000 pounds for compression members the excess
> shall be treated as a primary stress.

In the AASHO specification the permissible unit stresses
for structural carbon steel (1, p.145) to employ in the de-
sign of tension, compression and flexural members are govern-
ed by a base unit stress of 18,000 psi.

3. End conditions.

The ends of each test member in the tower are firmly connected to gusset plates which are also firmly connected to the other members framing into each of the joints. The effectiveness of these joints to resist rotation of the ends of the gusset plates is influenced by so many factors that the determination of the exact resistance does not lend itself to a rigorous mathematical analysis.

A reliable indication of this resistance, which will be referred to as the end condition, can be determined by a comparison of the strain data for a member in the tower with the corresponding strain data for the same member in the testing machine. The machine tests, in which various controlled end conditions were used, will serve as standards.

Since secondary bending exists only in the tower, and not in the machine, the tower data used for this comparison should not include any secondary strains. Since these secondary strains are a minimum near the center of the member strain data will be used for the four gages located on the center cross-section of the member. For each of the members these gages are numbered 23, 24, 25 and 26 as shown in Figure 6.

The first member for which the results are to be compared is D4 in compression, and Figures 13, 14, 15 and 16
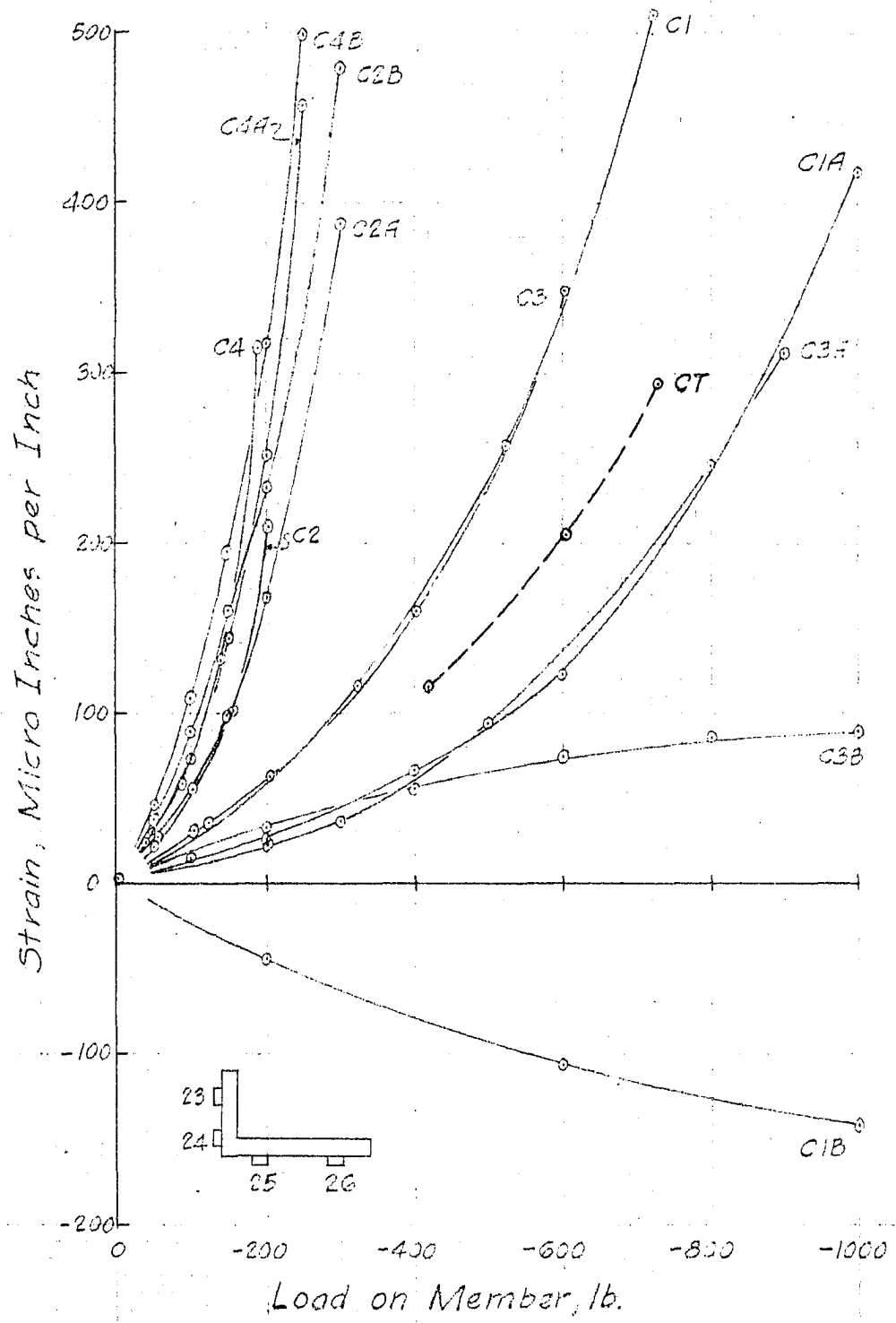
Figure 13. Member D4 in Compression, Load-Strain Curves
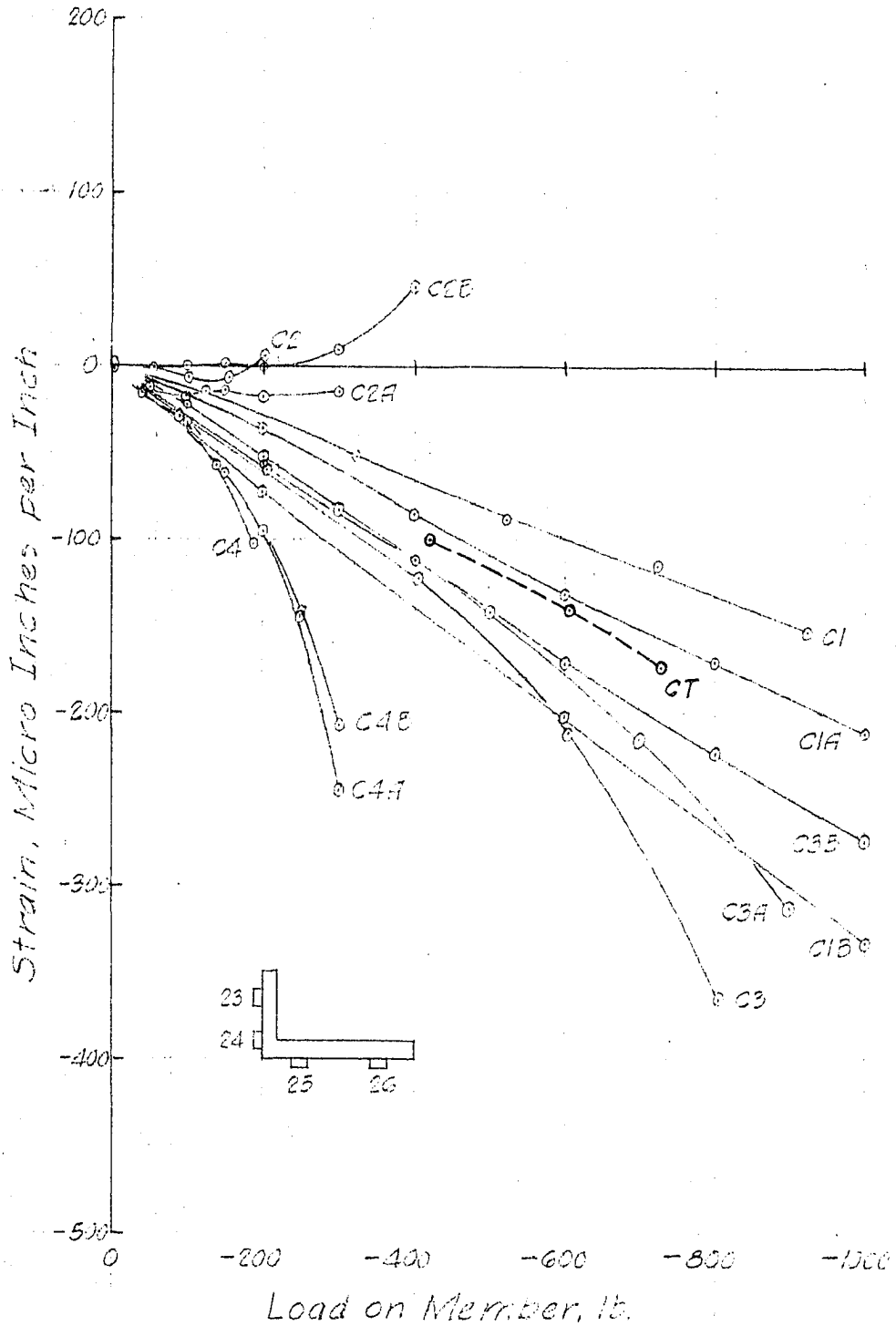for Gage 23.

Figure 14. Member D4 in Compression, Load-Strain Curves
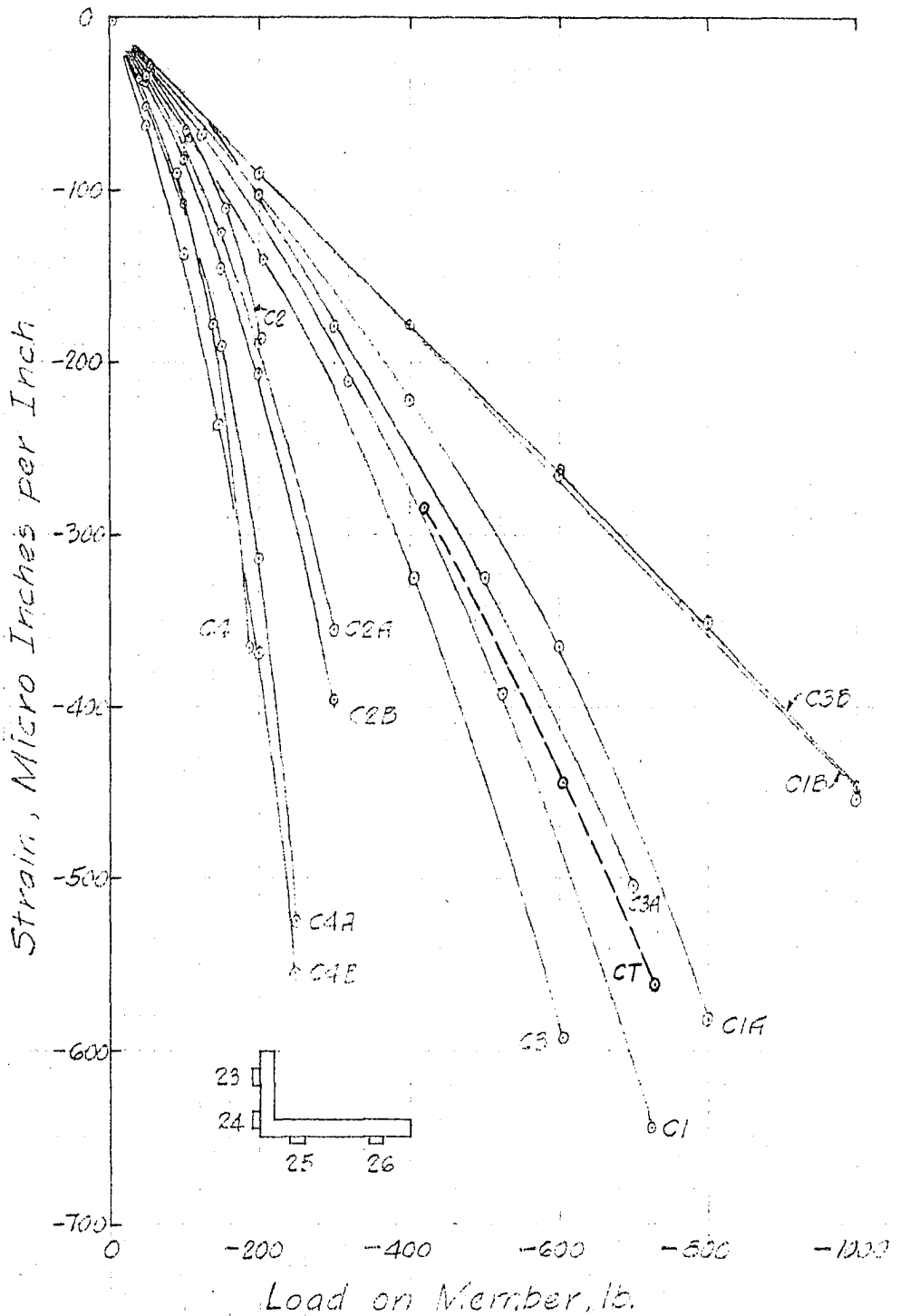for Gage 24.

Figure 15. Member D4 in Compression, Load-Strain Curves for Gage 25.
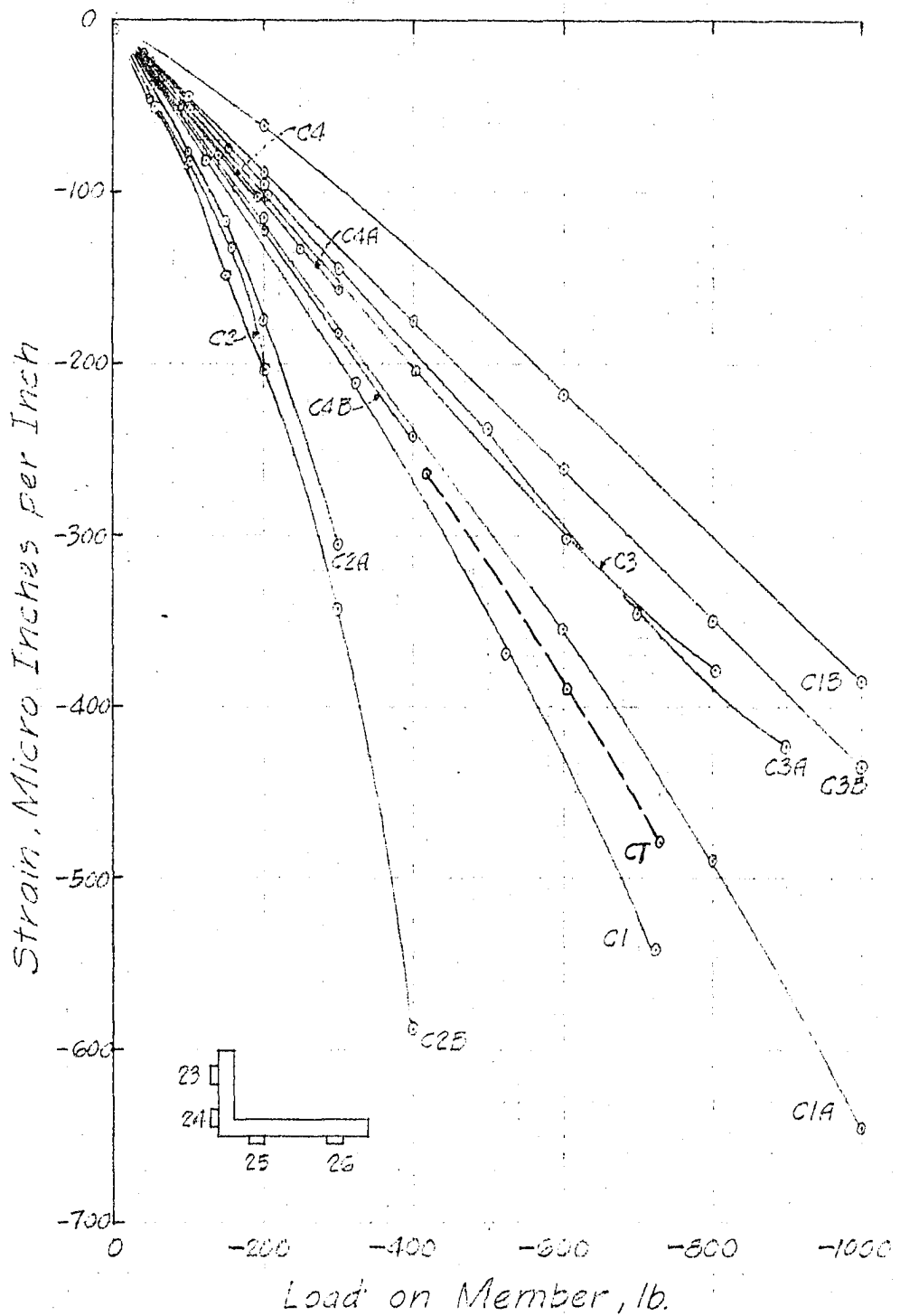
Figure 16. Member D4 in Compression, Load-Strain Curves

for Gage 26.

are the load-strain curves for this member. The curves marked CT are those for the tower test while all of the other curves are for the machine tests. Table 1 indicates the end conditions represented by the markings on these curves.

In all four of these figures it is apparent that the end conditions C2, C2A, C2B, C4, C4A and C4B cannot represent the end condition in the tower. The last point on each of these curves indicates the maximum increment of load the member would support in the machine. These maximum loads are far below those applied to the member in the tower as indicated by the CT curves. These tower loads caused no apparent distress in the member.

For gages 23 and 25 the curves for C1 and C3 are very close together, also C1A and C3A are close. In each case the CT curve lies between the two pairs C1, C3 and C1A, C3A. Therefore, the strain data for gages 23 and 25 indicates that the end conditions in the tower are between the pairs of conditions C1, C3 and C1A, C3A in the machine, but no differentiation can be made between them. For gages 24 and 26, however, the C1, C1A, C3 and C3A curves are distinctly different, and serve to identify further the end conditions in the tower. For gage 24 the CT curve lies between C1A and C3A, but closer to C1A. For gage 26 the CT curve lies between the C1 and C1A curves and quite removed from both C2 and C3A.

Table 1

Designation of End Conditions of Members in Testing Machine

Mark	Gusset Plates	End Condition

Compression Members

C1	1/8" Aluminum	
C1A	1/8" Steel	Rigid Fixed
C1B	1/4" Steel	
C2	1/8" Aluminum	
C2A	1/8" Steel	Hinge parallel to gussets
C2B	1/4" Steel	
C3	1/8" Aluminum	
C3A	1/8" Steel	Hinge normal to gussets
C3B	1/8" Steel	
C4	1/8" Aluminum	
C4A	1/8" Steel	Hinges parallel and normal to gussets
C4B	1/4" Steel	

Tension Members

T1	1/8" Aluminum	
T1A	1/8" Steel	Rigid Fixed
T1B	1/4" Steel	
T2	1/8" Aluminum	
T2A	1/8" Steel	Hinge parallel to gussets
T2B	1/4" Steel	
T3	1/8" Aluminum	
T3A	1/8" Steel	Hinge normal to gussets
T3B	1/4" Steel	
T4	1/8" Aluminum	
T4A	1/8" Steel	Hinges parallel and normal to gussets
T4B	1/4" Steel	

It would appear that the end condition for this member in the tower might be either C1, C1A, C3 or C3A, but for the gages where C3 and C3A are not close to C1 or C1A the CT curve lies closer to the curves C1 and C1A than to the curves C3 and C3A. Therefore, the end condition in the tower for the member D4 in compression is between C1 and C1A. Both of these conditions are rigid fixed. The only difference between them is the gusset plates used.

For the same member D4 in tension the curves for the same four gages appear in Figures 17, 18, 19 and 20. The curves marked TT are those for the tower test, while all of the other curves are for the machine tests. Table 1 indicates the end conditions represented by the markings on these curves.

The tension end conditions T2, T2A, T2B, T4, T4A and T4B are the same as the corresponding compression end conditions C2, C2A, C2B, C4, C4A and C4B. Since the compression results indicate that such end conditions do not occur in the tower, it is reasonable to arrive at the same conclusion for the corresponding tension end condition. On the basis of this conclusion the curves representing these conditions have been omitted in Figures 17, 18, 19 and 20.

For gage 23 the TT curve is closer to T1 than to any of the other curves, but T3 is not very far from it. For
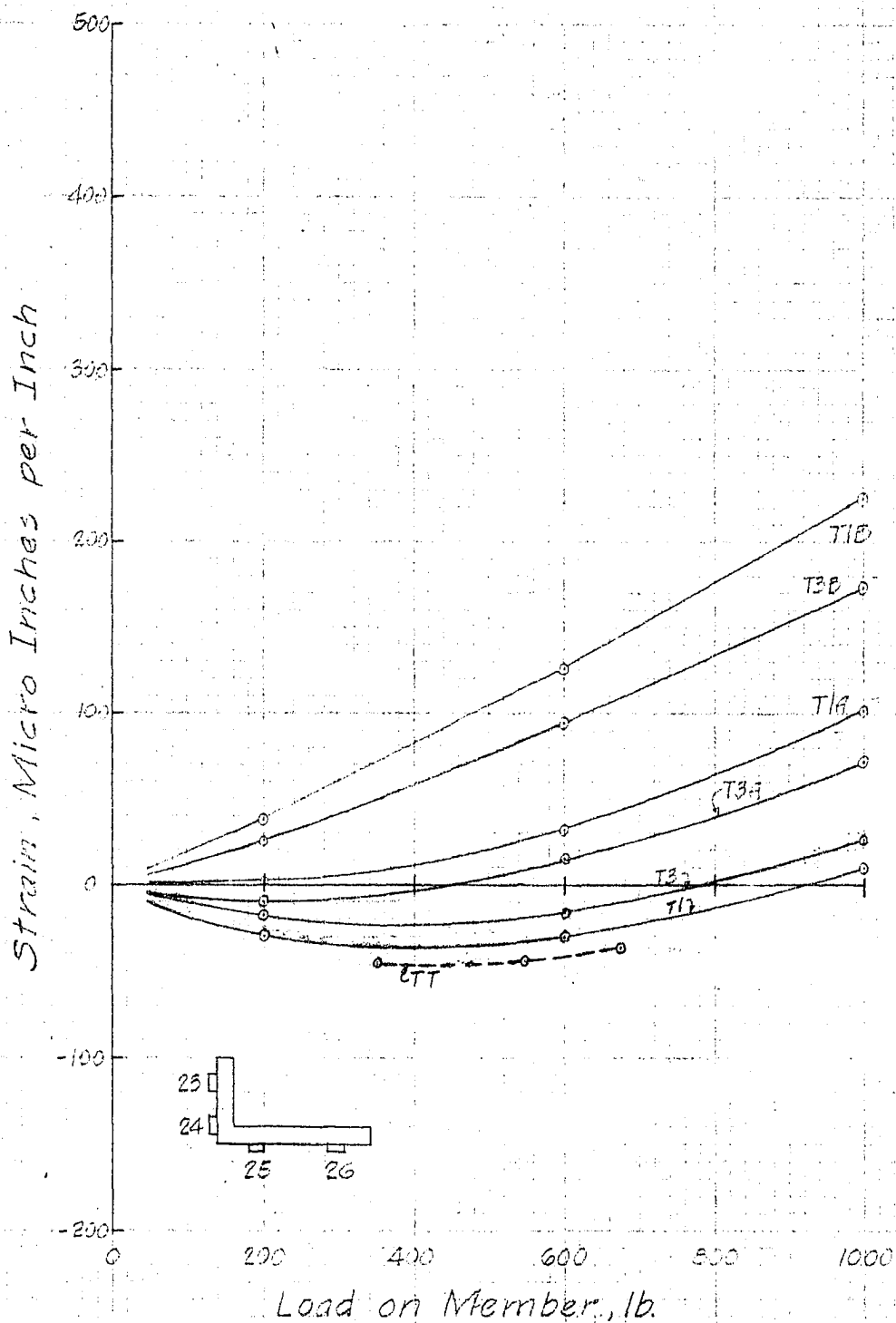
Figure 17. Member D4 in Tension, Load-Strain Curves for
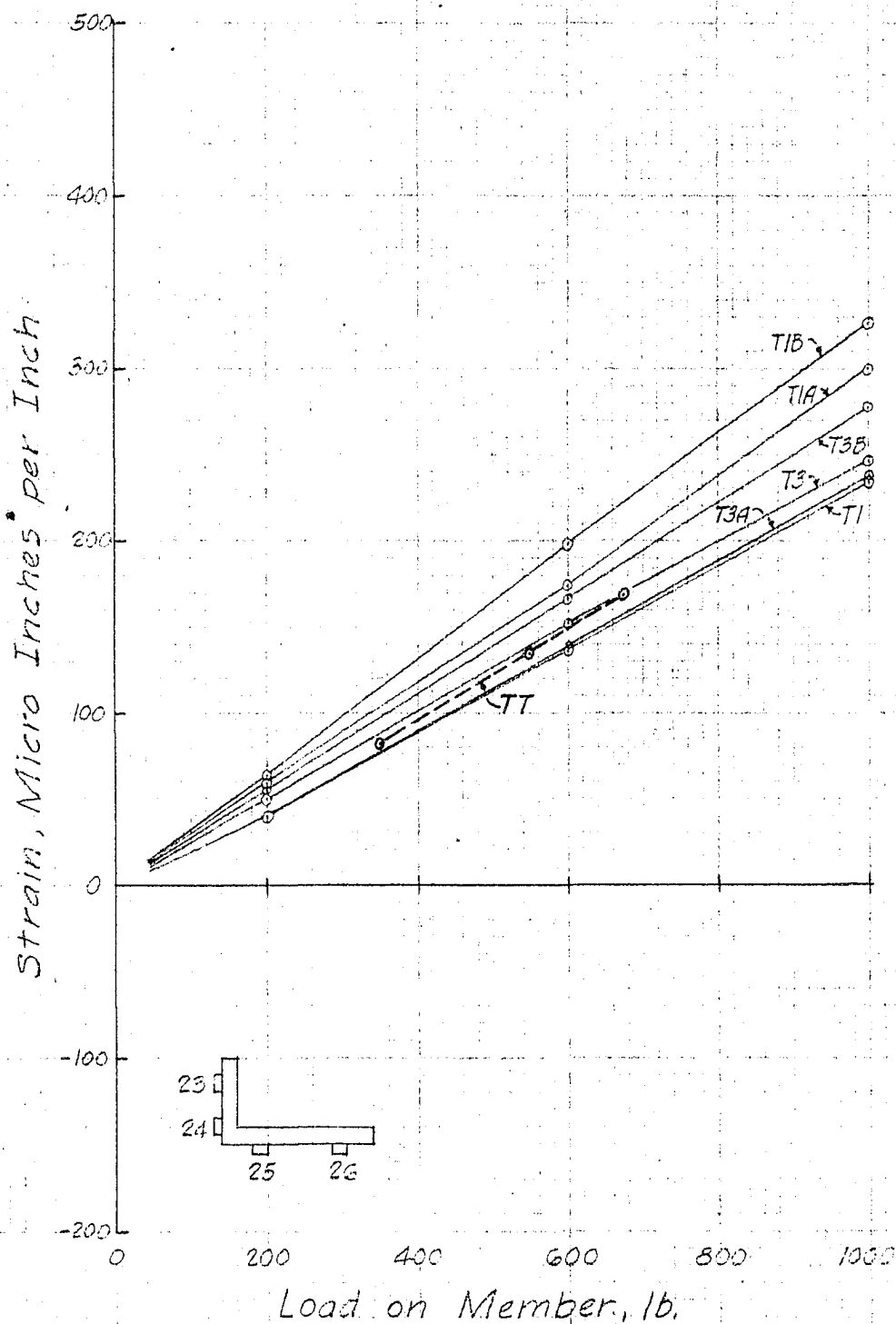Gage 23.

44

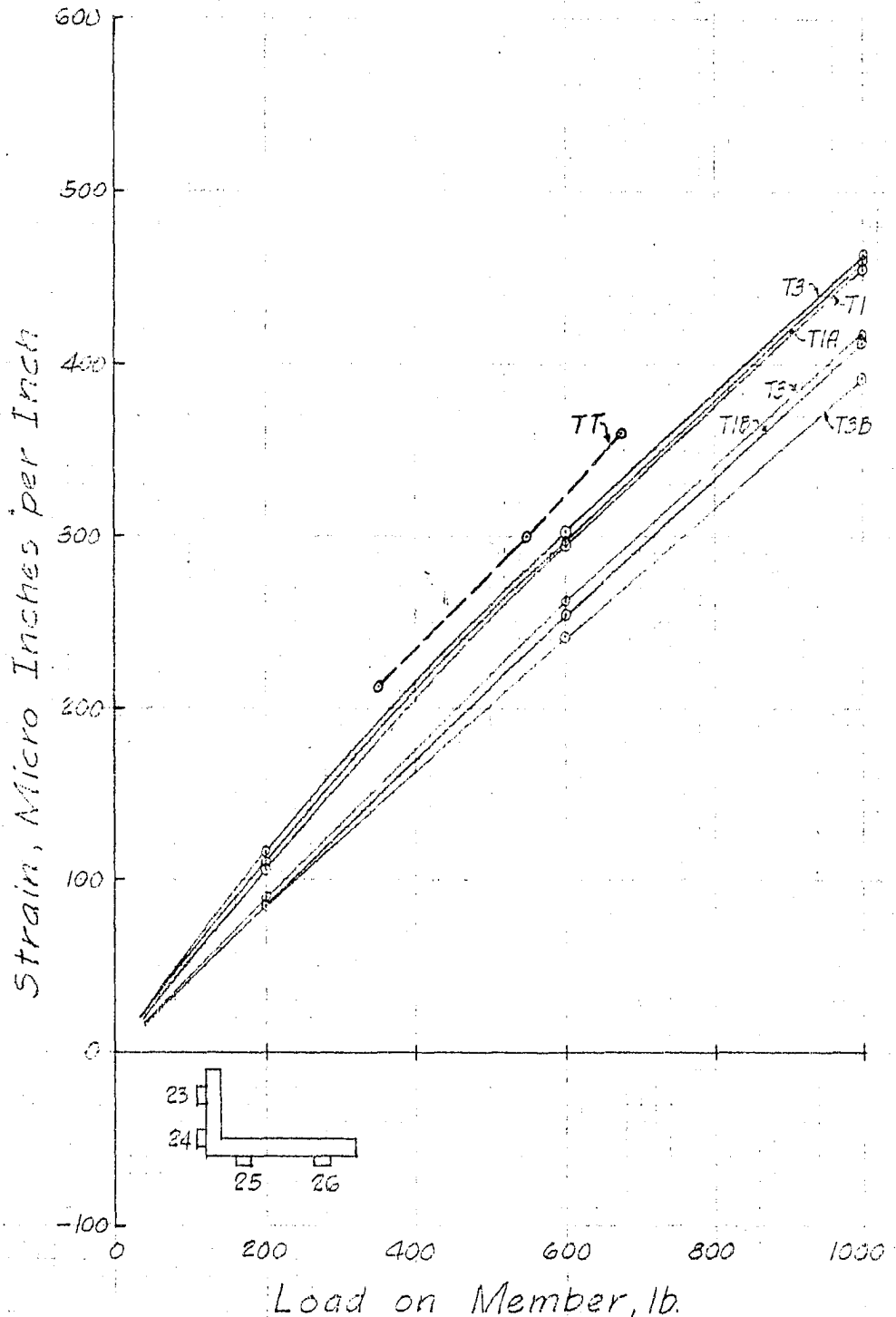Figure 18. Member D4 in Tension, Load-Strain Curves for

Gage 24.

Figure 19. Member D4 in Tension, Load-Strain Curves for
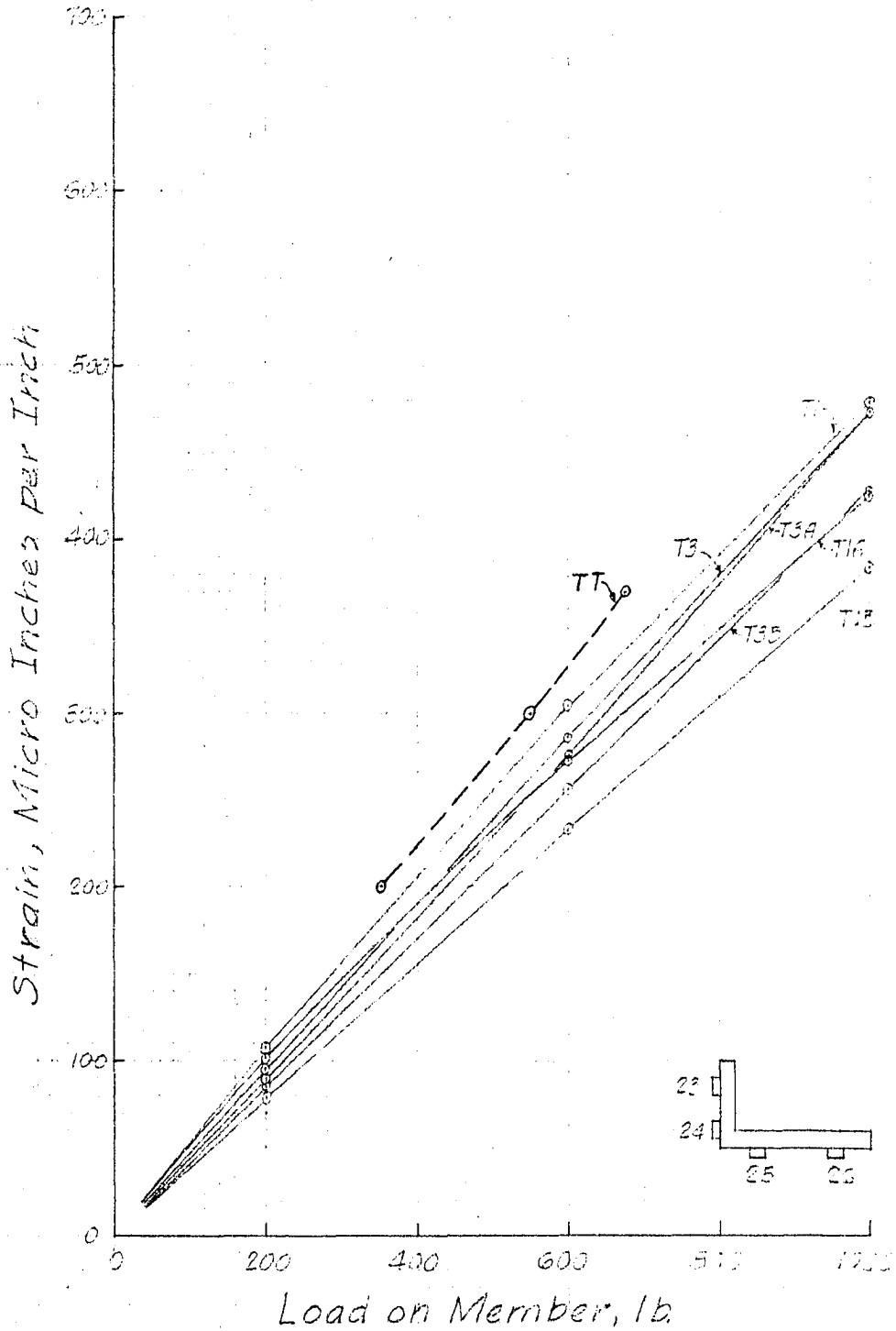Gage 25.

Figure 20. Member D4 in Tension, Load-Strain Curves for

Gage 26.

gage 24 the TT curve is between T1 and T3, but closer to T3.
For gage 25 the TT curve is close to both T1 and T3, which
are nearly the same. For gage 26 the TT curve is closest to
T1, but T3 is not very far from it. Therefore, the end con-
dition in the tower for member D4 in tension approaches both
end conditions T1 and T3 in the machine. Condition T1 is
rigid fixed, while T3 is hinged on an axis normal to the
plane of the gusset plates and rigid fixed normal to the
hinge.

Similar curves for members D5 and D7 in compression and
tension appear in Figure A11 to A26 inclusive. For both of
these members only the conditions C1, C2, C3, C4, T1, T2,
T3 and T4 were tested. These curves indicate that the end
condition in the tower for D5 in compression approaches both
C1 and C3, and for D5 in tension the end condition approaches
T1 and T3. For the member D7 in compression the end condition
approaches C1, and for D7 in tension the end condition
approaches T1 and T3.

For all three of the members both in tension and com-
pression the end conditions in the tower approach one of the
conditions C1, C1A, or T1 all of which are rigid fixed. In
addition, the end conditions of all of the tension members and
one of the compression members also approach the conditions
C3 or T3, which are hinged normal to the plane of the gusset
plates. In every case where the tower curves CT and TT are

close to both C1 and C3 or T1 and T3 these pairs of machine curves were close together. On the other hand, whenever the C1 and C3 or the T1 and T3 curves were not close together the tower curves were close to the C1 or T1 curves only.
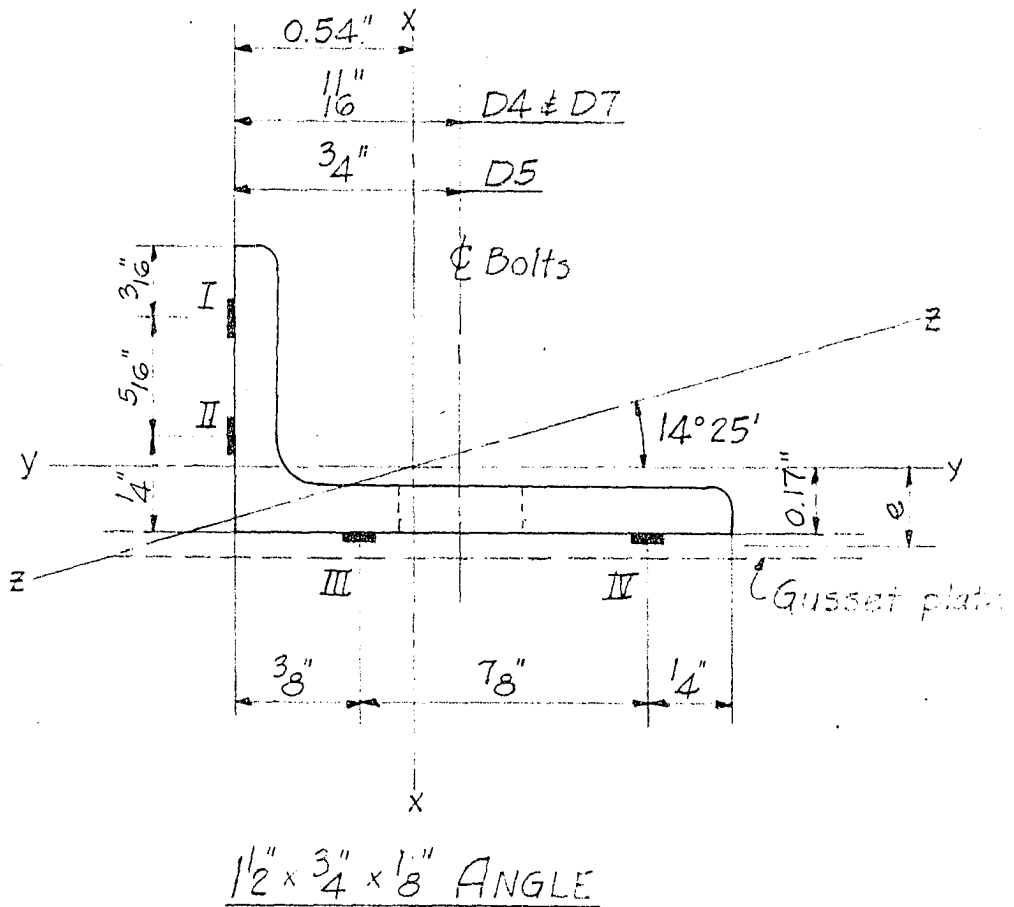
Therefore, it may be concluded that the ends of the gusset plates attached to the structure approach the rigid fixed condition.

4. <u>Elastic curvature</u>.

The test members in the tower are loaded at each end
with an axial load and a bending moment as previously dis-
cussed. Each bending moment may be resolved into two com-
ponent bending moments, one about the x-axis and the other
about the y-axis of the angle section, shown in Figure 21.

The member would bend about both the x-axis and the
y-axis producing an elastic curve whose deflection at any
point would have both x and y components. The magnitudes
of these deflection components are influenced by the re-
straint offered by the gusset plates to which the ends of
the member are joined.

The longitudinal strain curves for gage lines III and
IV for the tower test member D4 in compression and tension
appear in Figures 12 and A2. In each case strain curves
for three different loads are given. For each loading the
two curves cross each other near the center of the member
and diverge toward each end. For the tension member these
points of intersection are either at or very close to the
center of the member, but for the compression member they
are definitely toward one end of the member. The divergence
of each pair of curves has been attributed to the secondary
bending present in the member.

0.54" X

1 1/16" D4 & D7

3/4" D5

C Bolts

3/16"

5/16" I

II 14°25'

y y

1/4" 0.17" e

z z

III IV C Gusset plate

3/8" 7/8" 1/4"

X

1 1/2" × 3/4" × 1/8" ANGLE

SECTION PROPERTIES (12, p88)

Area, 0.27 in²

I_{xx}, 0.061 in⁴

r_{xx}, 0.48 in

I_{yy}, 0.010 in⁴

r_{yy}, 0.20 in

r_{zz}, 0.16 in

Figure 21. Cross-Section and Section Properties of Test

Members.

For the tension member, if there was no secondary bending the primary curves for gage lines III and IV would be practically the same. However, for the compression member, if there was no secondary bending the primary curves for the two gage lines would be of entirely different shapes. The primary curve for gage line IV would approach a straight line while the curve for gage line III would have a very definite curvature. This lack of curvature for gage line IV might be explained as follows. Gage line IV is near the unsupported edge of the long leg of the angle and is in compression. This unsupported edge is waving or buckling slightly and relieving itself of some of the load. If the strain curves for each of the gage lines IV were ideally curved similar to those for gage line III the points of intersection of these pairs of curves would be closer to the center of the member than they are now.

Similar longitudinal strain curves for gage lines III and IV for the tower test members D5 and D7 in compression and tension appear in Figures A4, A6, A8 and A10, in Appendix A. These curves are all similar to the corresponding curves for the member D4.

It is indicated by the above observations that the primary bending of the angle member about its x-axis is negligible. If there was no bending about the x-axis of the angle then the primary strains along gage lines III and

and IV would be equal at every cross-section. However, when
the secondary strains are added to these primary strains
there would be only one cross-section near the center of the
member when the two combined strains would be equal. Since
the curves for all of the tower tests indicate such a con-
dition it can be assumed that all of the bending of the
angle member is about its y-axis.

Batho's (4, p. 145) strain curves for the center cross-
section of the single-angle tension members in the testing
machine indicate practically a constant strain across the
attached leg. Templin, Hartmann and Hill's unit stress dis-
tribution curves (5, p. 37) for the single-angle members in
a truss indicate a point near the center of each member
where the unit stress would be constant across the attached
leg. Both of these results would indicate in a way similar
to that of this study that practically no primary bending exists
about an axis normal to the plane of the gusset plates. In
both of these cases the angles used were equal leg angles
indicating that the type of bending observed in this study does
not depend on a particular type of angle.

B. Compression Members in Tower.

Having established some of the basic factors affecting the behavior of the compression members in the tower it is possible to develop a practical method of design for such members.

1. Development of theory.

On the basis of previously observed data defining the behavior of the test members in the tower a theoretical analysis will be made for the type of compression members tested in this investigation. This analysis will be based on two assumptions resulting from these observations. The first is that the ends of the gusset plates joined to the structure are rigid fixed, and the second is that no bending of the member occurs about the axis of the member normal to the plane of the gusset plates.

In addition it is assumed that the member is symmetrical about its center-line and that the bolted connections of the gusset plates to the member are rigid and located at the centroid of the bolt pattern. Also included are all of the usual assumptions made in the development of the basic theory used. Figure 22 shows the assumed member as a free-body diagram and its elastic curve with the necessary notation.
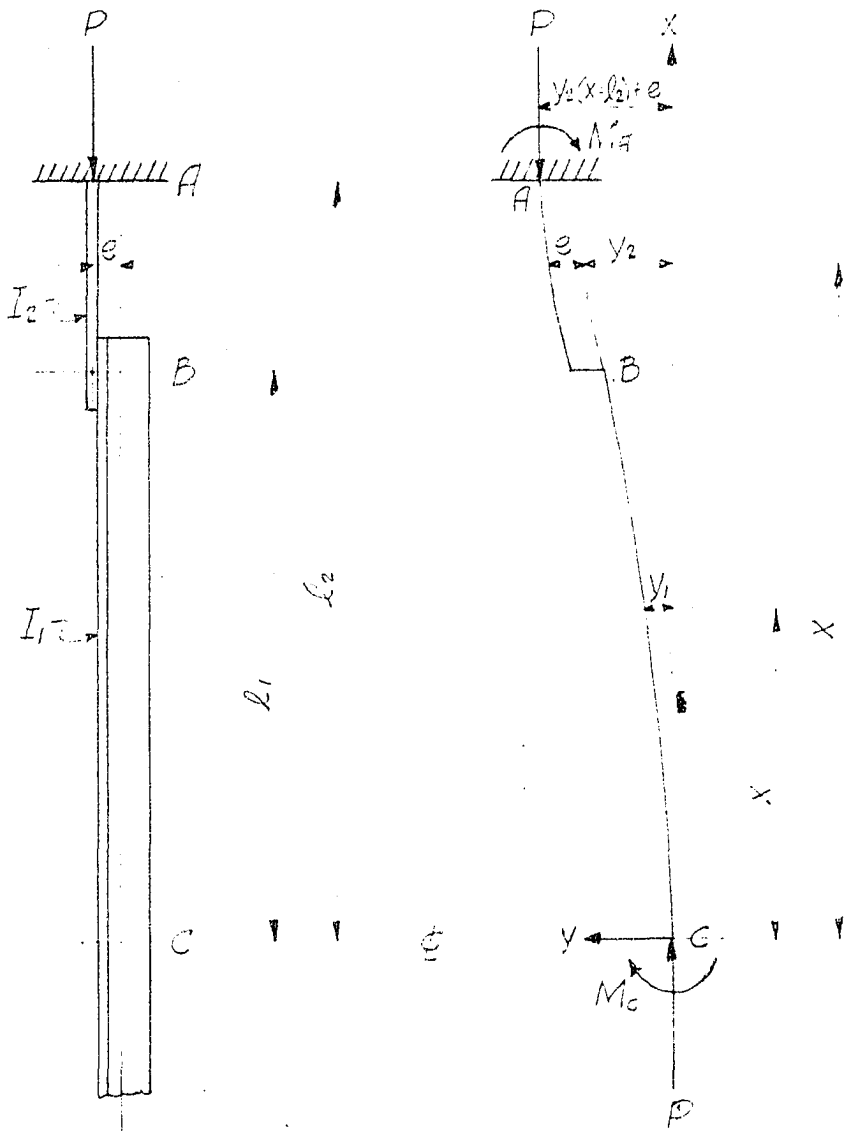
Figure 22. Typical Compression Member

For small deflections the differential equation for the beam is:

$$EI\frac{d^2y}{dx^2} = M \tag{1}$$

The general expressions for bending moment are:

$$M_1 = M_c - Py_1, \quad \text{for } x = 0 \text{ to } \ell_1 \tag{2}$$

$$M_2 = M_c - P(y_2 + e), \quad \text{for } x = \ell_1 \text{ to } \ell_2 \tag{3}$$

Substituting (2) into equation (1) yields the differential equation:

$$\frac{d^2y_1}{dx^2} + k_1^2 y_1 = q_1 \tag{4}$$

where $k_1^2 = \dfrac{P}{EI_1}$ and $q_1 = \dfrac{M_c}{EI_1}$

The general solution of equation (4) is:

$$y_1 = C_1 \cos k_1 x + C_2 \sin k_1 x + \frac{M_c}{P} \tag{5}$$

Substituting (3) into equation (1) yields the differential equation:

$$\frac{d^2y_2}{dx^2} + k_2^2 y_2 = q_2 - k_2^2 e \tag{6}$$

where $k_2^2 = \dfrac{P}{EI_2}$ and $q_2 = \dfrac{M_c}{EI_2}$

The general solution of equation (6) is:

$$y_2 = C_3 \cos k_2 x + C_4 \sin k_2 x + \frac{M_c}{P} - e \tag{7}$$

The boundary conditions for the member are:

(I) $y_1 = 0$ when $x = 0$

(II) $\dfrac{dy_1}{dx} = 0$ when $x = 0$

(III) $y_1 = y_2$ when $x = \ell_1$

(IV) $\dfrac{dy_1}{dx} = \dfrac{dy_2}{dx}$ when $x = \ell_1$

(V) $\dfrac{dy_2}{dx} = 0$ when $x = \ell_2$

Applying boundary conditions (I) and (II) to equation (5) yields:

$$y_1 = \frac{Mc}{P}(1 - \cos k_1 x) \qquad (8)$$

Applying boundary conditions (III), (IV), and (V) to equation (7) yields:

$$y_2 = A \cos k_2 x + B \sin k_2 x + \frac{Mc}{P} - e \qquad (9)$$

where:

$$A = \frac{k_1 Mc \sin k_1 \ell_1}{k_2 P \tan k_2 \ell_2 \left(\cos k_2 \ell_1 - \dfrac{\sin k_2 \ell_1}{\tan k_2 \ell_2} \right)}$$

and:

$$B = \frac{k_1 Mc \sin k_1 \ell_1}{k_2 P \left(\cos k_2 \ell_1 - \dfrac{\sin k_2 \ell_1}{\tan k_2 \ell_2} \right)}$$

Equating equations (8) and (9) results in the following equation for the bending moment at the center-line of the member:

$$M_{ic} = \cfrac{Pe}{\dfrac{k_1}{k_2}\left[\dfrac{\sin k_1 l_1 \cos k_2 l_1}{\tan k_2 l_2 \left(\cos k_2 l_1 - \dfrac{\sin k_2 l_1}{\tan k_2 l_2}\right)} + \right.}$$

$$\cfrac{}{\left. + \dfrac{\sin k_1 l_1 \sin k_2 l_1}{\cos k_2 l_1 - \dfrac{\sin k_2 l_1}{\tan k_2 l_2}}\right] + \cos k_1 l_1} \qquad (10)$$

For a particular member the bending moment at any section can be computed by substituting equation (10) in equation (2) or (3). The deflection of the member can be computed at any point by substituting equation (10) in equation (8) or (9).

2. <u>Comparison of derived theory with results observed in</u>
   <u>testing machine.</u>

To verify equation (10) experimentally the results of
the machine tests Cl for the members will be used. Machine
tests are used in preference to tower tests for three reasons:

1. Secondary bending does not develop and this
    bending was not considered in the development
    of equation (10).

2. The members and their end connections are
    symmetrical while in the tower the gusset
    plates at each end of a member are not identical.

3. The effective lengths and moments of inertia of
    the gusset plates are defined while in the tower
    they are practically indeterminate.

The machine condition Cl is used because its controlled con-
ditions closely satisfy the assumptions on which equation
(10) is based, and data are available for all three of the
test members.

To compare the computed results from equation (10) with
the observed data from condition Cl for each of the members,
longitudinal strain curves for the four gage lines I, II, III
and IV are used, and these curves appear in Figures 23, 24 and
25. The data for the curves representing equation (10) were
computed on the basis of the modulus of elasticity of the

Figure 23. Member D4 in Compression, Comparison of Equation
(10) with Condition C1 for Load of 731 lb.

61
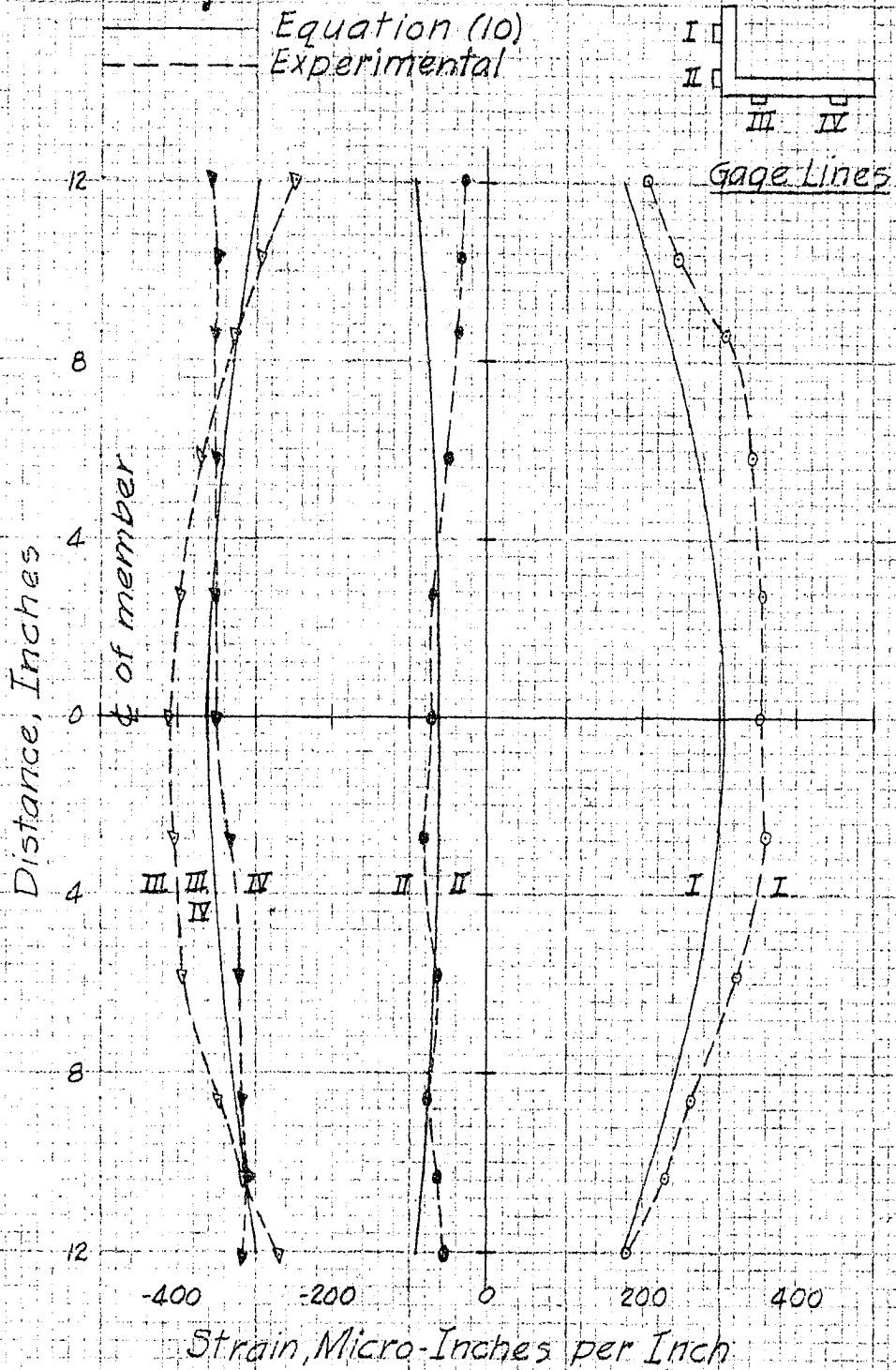
Figure 24. Member D5 in Compression, Comparison of Equation
(10) with Condition C1 for Load of 449 lb.
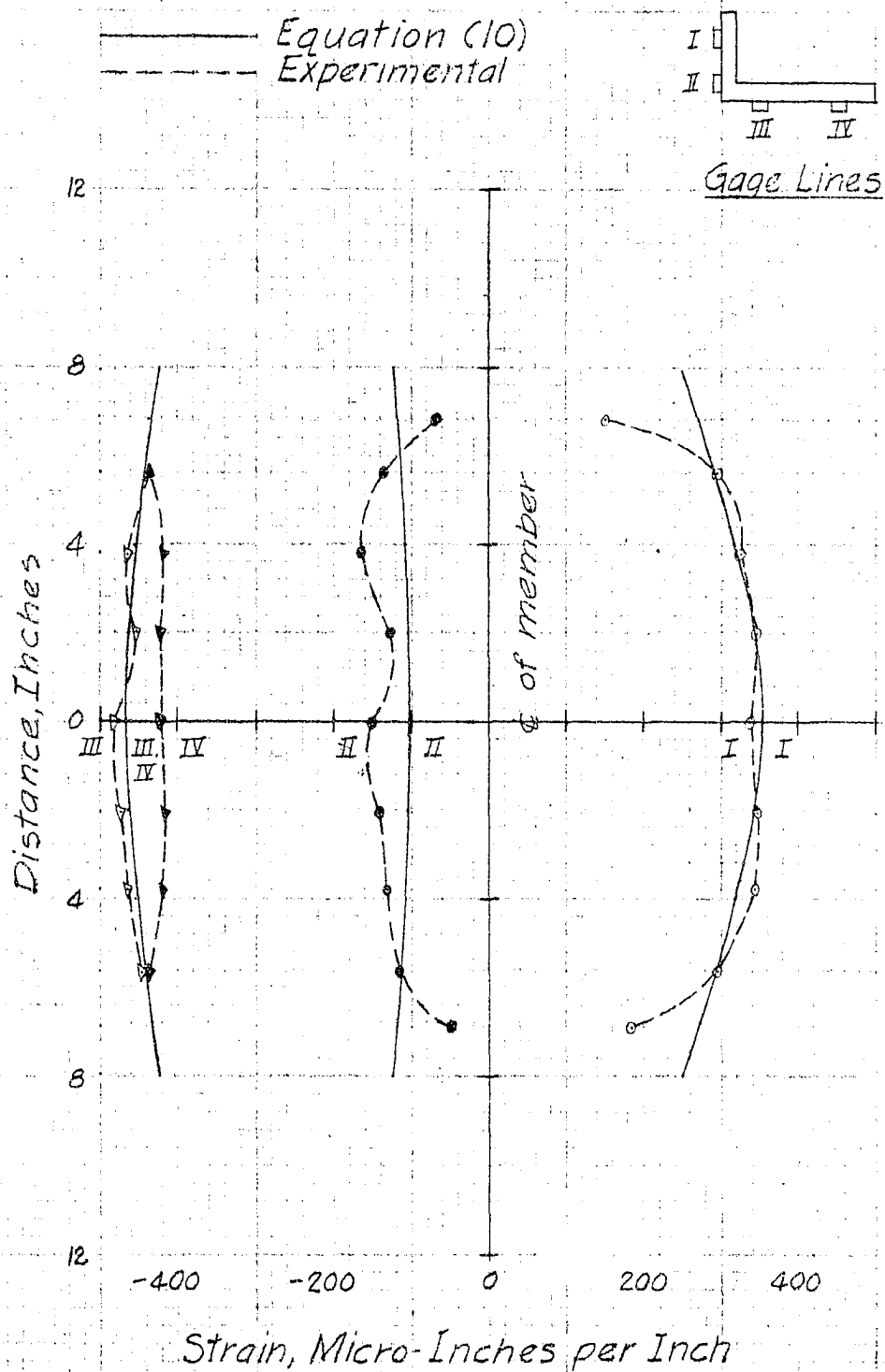
Figure 25. Member D7 in Compression, Comparison of Equation (10) with Condition C1 for Load of 632 lb.

material in the test members as determined in Appendix C.

An examination of the curves plotted for the derived equation (10) show very close agreement with those plotted from the data obtained on tests with single-angle members in the testing machine. Therefore, the derived equation (10) is verified for the controlled conditions of this portion of the investigation.

3. <u>Effective length of members</u>.

The compression member used in this investigation has two inflection points as indicated by the general shape of its elastic curve, shown in Figure 22. The length of the member between these inflection points is called the effective length. If this effective length can be established then the analysis of the compression member can be simplified. The member can be considered as a hinged end compression member of a length equal to this effective length.

At an inflection point the bending moment is zero and therefore from equation (1), at that point:

$$\frac{d^2y}{dx^2} = 0 \qquad (11)$$

If the inflection point is between C and B in Figure 22 then x is between zero and ℓ_1 and equation (8) can be used. Then equation (11) becomes:

$$\frac{d^2y_1}{dx^2} = 0 \qquad (12)$$

Differentiating equation (8) twice and substituting in equation (12) gives:

$$\frac{d^2y_1}{dx^2} = \frac{M_c}{P} k_1^2 \cos k_1 x = 0 \qquad (13)$$

Since M_c and k_1 can be zero only when P is zero, then:

$$\cos k_1 x = 0 \qquad (14)$$

Equation (14) will be satisfied when:

$$k_1 x = \frac{n\pi}{2} \qquad (15)$$

where $n = 1, 3, 5 \cdots$ Then from equation (15):

$$x = \frac{n\pi}{2k_1} \qquad (16)$$

and substituting for k_1, its value $\sqrt{\frac{P}{EI_1}}$ gives:

$$x_e = \frac{n\pi}{2}\sqrt{\frac{EI_1}{P}} \qquad (17)$$

where x_e is one-half of the effective length of the member. Then from equation (17):

$$P = \frac{n^2 \pi^2 E I_1}{4 x_e^2} \qquad (18)$$

where $n = 1, 3, 5 \cdots$.

67

Equation (18) is identical in form to the Euler equation for the critical load on a concentrically loaded strut with hinged ends where X_e is one-half the length of the strut. The load P is inversely proportional to X_e and, therefore, the smallest load producing general buckling of the member from A to B (Figure 22) as a unit results when $n = 1$ and $X_e = l_1$. If the load P is less than this critical load then X_e is greater than l_1. Therefore, the inflection point cannot be between B and C unless the load on the member BC is greater than its critical load of the least magnitude.

In Figure 22 the section of the member from A to B is identical in form to the section from B to C, and similarly the inflection point cannot be at any point between A and B unless the load on the member AB is greater than its critical load of the least magnitude.

Then, if neither the angle member BC nor the gusset plate AB is loaded beyond its respective critical loads the inflection point must lie in the eccentric connection at B between the centroid of the angle and the center of the gusset plate. Therefore, the effective length of the compression member is the distance between the centroids of the bolted connections between the gusset plates and the angle member, providing neither the angle member nor the gusset plates are loaded beyond their critical loads of the least magnitude.

4. <u>Development of practical method of design.</u>

Equation (10), which was developed for compression members of the type tested in this investigation, is of value for theoretical investigations, determination of limits and development of practical methods of design. However, it is of very little direct value to the practical designer.

The equation is so involved and its application so time consuming that the practical designer would not bother to use it. In addition, it is necessary to know, or assume, the effective length and the moment of inertia of the gusset plate at each end of the member. These factors are indeterminate and even difficult to assume for gusset plates such as those on the test members in the tower (Figures 3, 4 and 5). Also, the effective stiffness of such a gusset plate changes as the load is applied to the structure. A gusset plate at the end of a compression member also has other members connected to it. Tension members will increase, while other compression members will decrease the effective stiffness of the gusset plate as the load on each of these members is increased.

Therefore, it is highly desirable to develop a method of design which can be readily applied by the designer to his practical problems. Such a method will be developed on the basis of additional simplifying assumptions based on the

theoretical and experimental results of this investigation.

The design of compression members subjected to direct and bending loads necessitates the use of a trial and error procedure where a member is selected and then checked to see whether it can safely support the design load. The maximum theoretical load for such a member is limited either by the critical load, based on general buckling of the member as a unit, or by a controlling unit stress at some point. This controlling unit stress may be based on any of the following: a) the yield strength of the material; b) the lateral buckling of the member; and, c) the localized failure where a relatively small element of a member fails due to compression or buckling. The maximum theoretical load is reduced to determine the safe working load for the member. To accomplish this reduction a factor of safety is applied on the basis of prevailing conditions to be satisfied by the compression member.

To determine the critical load the angle member can be assumed to be a concentrically loaded hinged end strut of a length equal to its effective length. The effective length has been established in this study as the distance between the centroids of the two connections of the angle and the gusset plates. On the basis of this assumption the critical load can be determined with Euler's well known formula for such a member.

As previously determined herein, the two inflection points for this member are located in the connections between the centroid of the angle and the centers of the gusset plates. Therefore, if these points could be definitely located the angle member could be considered as an eccentrically loaded hinged end compression member. The length of the member would be the effective length and the eccentricities would be the distance from the centroid of the angle to the points of inflection.

The position of these points can be determined with equations (8) and (10). An examination of these equations, however, shows that the location is a function of the following: P, e, l_1, l_2, I_1, I_2, E_1 and E_2. Therefore, the inflection point is not fixed in position even for a particular member, but moves across the connection as the load varies. When the load on the member equals the critical load of either the angle or the gusset plate the point of inflection will be at the end of the connection joined to the critical element. Therefore, for a member which is not loaded beyond the critical load of either of its elements, the limits on the position of the inflection point are the centroid of the angle and the center of the gusset plates. In the design of such a member a factor of safety would be applied so that the design load on the member would not even

approach the critical load. Then, for a member subjected
to design loads, the maximum distance the point of inflction
can be from the centroid of the angle is less than the dis-
tance to the center of the gusset plates.

On the basis of the foregoing discussion, it can be
conservatively assumed that the inflction points are at
the center of the gusset plates, and the eccentricity at
each end of the member is equal to the distance from the
center of the gusset plate to the centroid of the angle.
For this assumed member the stresses can be computed by
the use of several available formulas for eccentrically
loaded compression members, the best known of which is
the secant formula.

A comparison of the computed strains, based on the
foregoing assumptions and the secant formula, with the
observed strains for the member D4 in compression in the
testing machine, condition C1, and in the tower are shown
in Figures 26 and 27. Similar curves for the members D5
and D7 appear in Figures A27 to A30 inclusive in Appendix
A. In all cases the results of the secant formula are
conservative in comparison with the observed results. This
reserve strength becomes available to compensate for the
uncertainties in determining the magnitude of the loads and
the effects of rolling and fabricating the members.

72
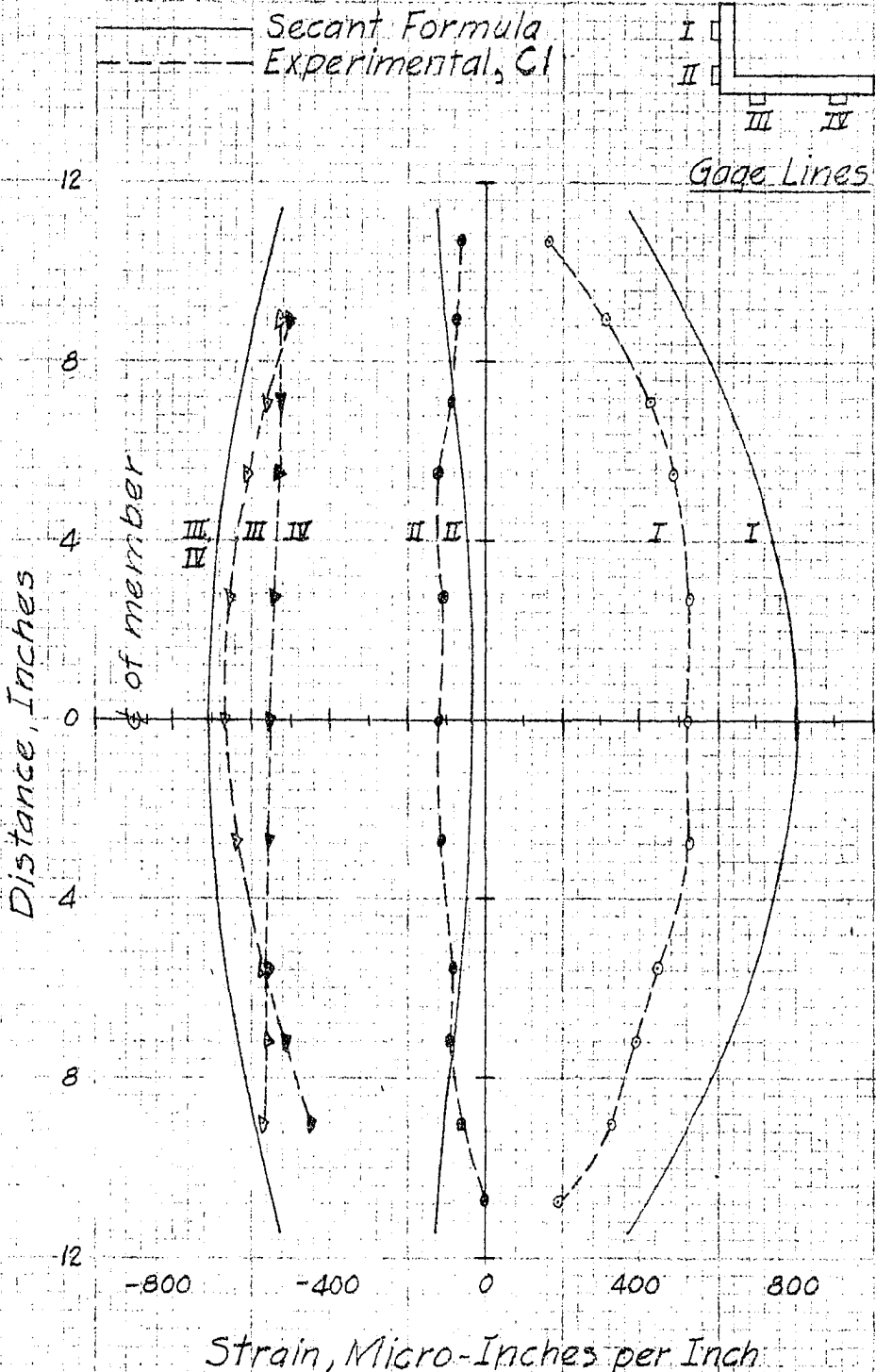
Figure 26. Member D4 in Compression, Comparison of Secant
Formula and Condition C1 for Load of 731 lb.
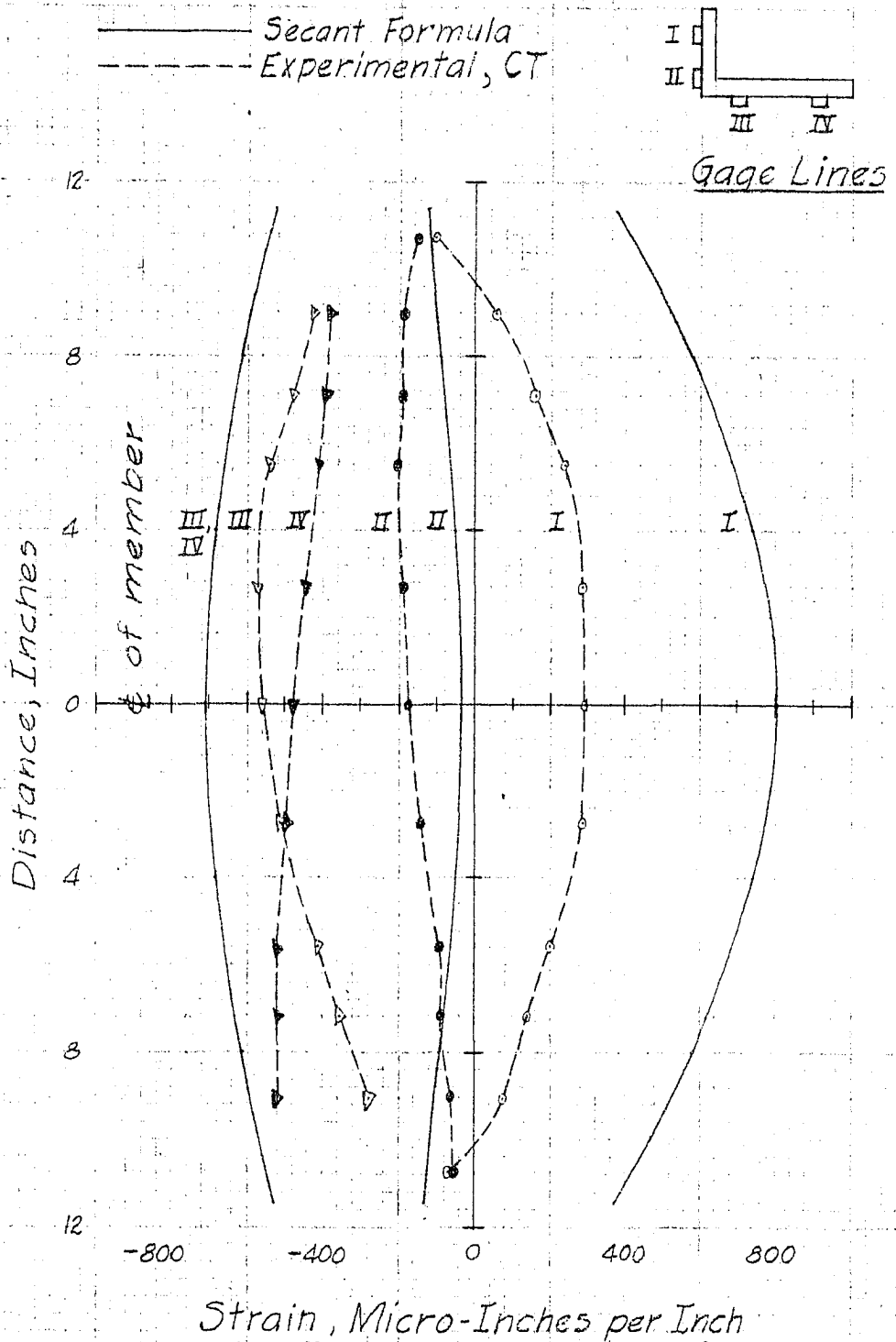
Figure 27. Member D4 in Compression, Comparison of Secant Formula with Condition CT for Load of 731 lb.

C. Tension Members in Tower.

Having established some of the basic factors affecting
the behavior of the tension members in the tower it is possible
to develop a practical method of design for such members.

1. Development of theory.

On the basis of the observed data defining the behavior
of the test members in the tower a theoretical analysis will
be made for the type of tension member tested in this investi-
gation. This analysis will be based on the same assumptions
as those previously made for the analysis of the compression
members. Figure 28 shows the assumed member and its elastic
curve with the necessary notation.
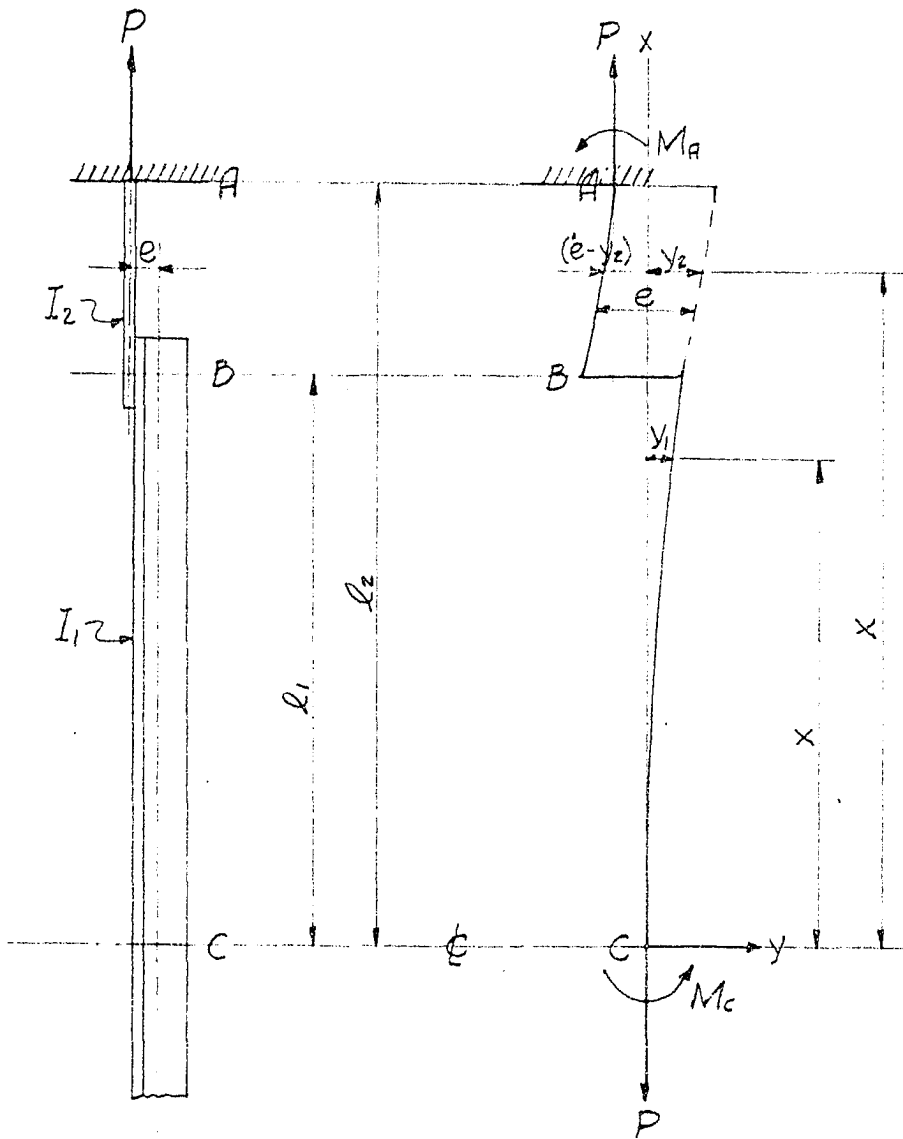
Figure 28.   Typical Tension Member.

For small deflections the differential equation for the beam is:

$$EI\frac{d^2y}{dx^2} = M \qquad (19)$$

The general expressions for bending moment are:

$$M_1 = M_c + Py_1 \quad , \text{ for } x = 0 \text{ to } \ell_1 \qquad (20)$$
$$M_2 = M_c + P(e-y_1), \text{ for } x = \ell_1 \text{ to } \ell_2 \qquad (21)$$

Substituting (20) into equation (19) yields the differential equation:

$$\frac{d^2y_1}{dx^2} - k_1^2 y_1 = q_1 \qquad (22)$$

where $k_1^2 = \dfrac{P}{EI_1}$ and $q_1 = \dfrac{M_c}{EI_1}$

The general solution of equation (22) is:

$$y_1 = C_1 \cosh k_1 x + C_2 \sinh k_1 x - \frac{M_c}{P} \qquad (23)$$

Substituting (21) into equation (19) yields the differential equation:

$$\frac{d^2y_2}{dx^2} - k_2^2 y_2 = q_2 - k_2^2 e \qquad (24)$$

where $k_2^2 = \dfrac{P}{EI_2}$ and $q_2 = \dfrac{M_c}{EI_2}$

The general solution of equation (24) is:

$$y_2 = C_3 \cosh k_2 x + C_4 \sinh k_2 x - \frac{M_c}{P} + e \qquad (25)$$

The boundary conditions for the members are:

(I) $y_1 = 0$ when $x = 0$

(II) $\dfrac{dy_1}{dx} = 0$ when $x = 0$

(III) $y_1 = y_2$ when $x = \ell_1$

(IV) $\dfrac{dy_1}{dx} = \dfrac{dy_2}{dx}$ when $x = \ell_1$

(V) $\dfrac{dy_2}{dx} = 0$ when $x = \ell_2$

Applying boundary conditions (I) and (II) to equation (23) yields:

$$y_1 = \frac{M_c}{P}(\cosh k_1 x - 1) \qquad (26)$$

Applying boundary conditions (III), (IV), and (V) to equation (25) yields:

$$y_2 = C \cosh k_2 x + D \sinh k_2 x - \frac{M_c}{P} + e \qquad (27)$$

where:

$$C = \frac{k_1 M_c \sinh k_1 \ell_1}{k_2 P \tanh k_2 \ell_2 \left(\dfrac{\sinh k_2 \ell_1}{\tanh k_2 \ell_2} - \cosh k_2 \ell_1 \right)}$$

and:

$$D = -\frac{k_1 M_c \sinh k_1 \ell_1}{k_2 P \left(\dfrac{\sinh k_2 \ell_1}{\tanh k_2 \ell_2} - \cosh k_2 \ell_1 \right)}$$

Equating equations (26) and (27) results in the following equation for the bending moment at the center-line of the member:

$$M_c = \cfrac{Pe}{\cfrac{k_1}{k_2}\left[\cfrac{\sinh k_1\ell_1 \sinh k_2\ell_1}{\cfrac{\sinh k_2\ell_1}{\tan k_2\ell_2} - \cosh k_2\ell_1} - \right.}$$

Con't on next line

$$\left. - \cfrac{\sinh k_1\ell_1 \cosh k_2\ell_1}{\tanh k_2\ell_2\left(\cfrac{\sinh k_2\ell_1}{\tanh k_2\ell_2} - \cosh k_2\ell_1\right)}\right] + \cosh k_1\ell_1} \tag{28}$$

For a particular member the bending moment at any section can be computed by substituting equation(28)in equation (20) or (21). The deflection of the member can be computed at any point along the length of the member by substituting equation (28) in equation (26) or (27).

2. <u>Comparison of derived theory with results observed in</u>

<u>testing machine</u>.

To verify equation (28) experimentally the results of
the machine tests T1 for the members will be used. Machine
tests are again used for the same reasons similar tests were
used for the compression members. The machine condition T1
is used because its controlled conditions closely satisfy
the assumptions on which equation (28) is based, and data
are available for all three of the test members.

To compare the computed results from equation (28) with
the observed data from condition T1 for each of the members,
longitudinal strain curves for the four gage lines I, II,
III and IV are used, and these curves appear in Figures 29,
30 and 31. The data for the curves representing equation
(28) were computed on the basis of the modulus of elasticity
of the material in the test members as determined in
Appendix C.
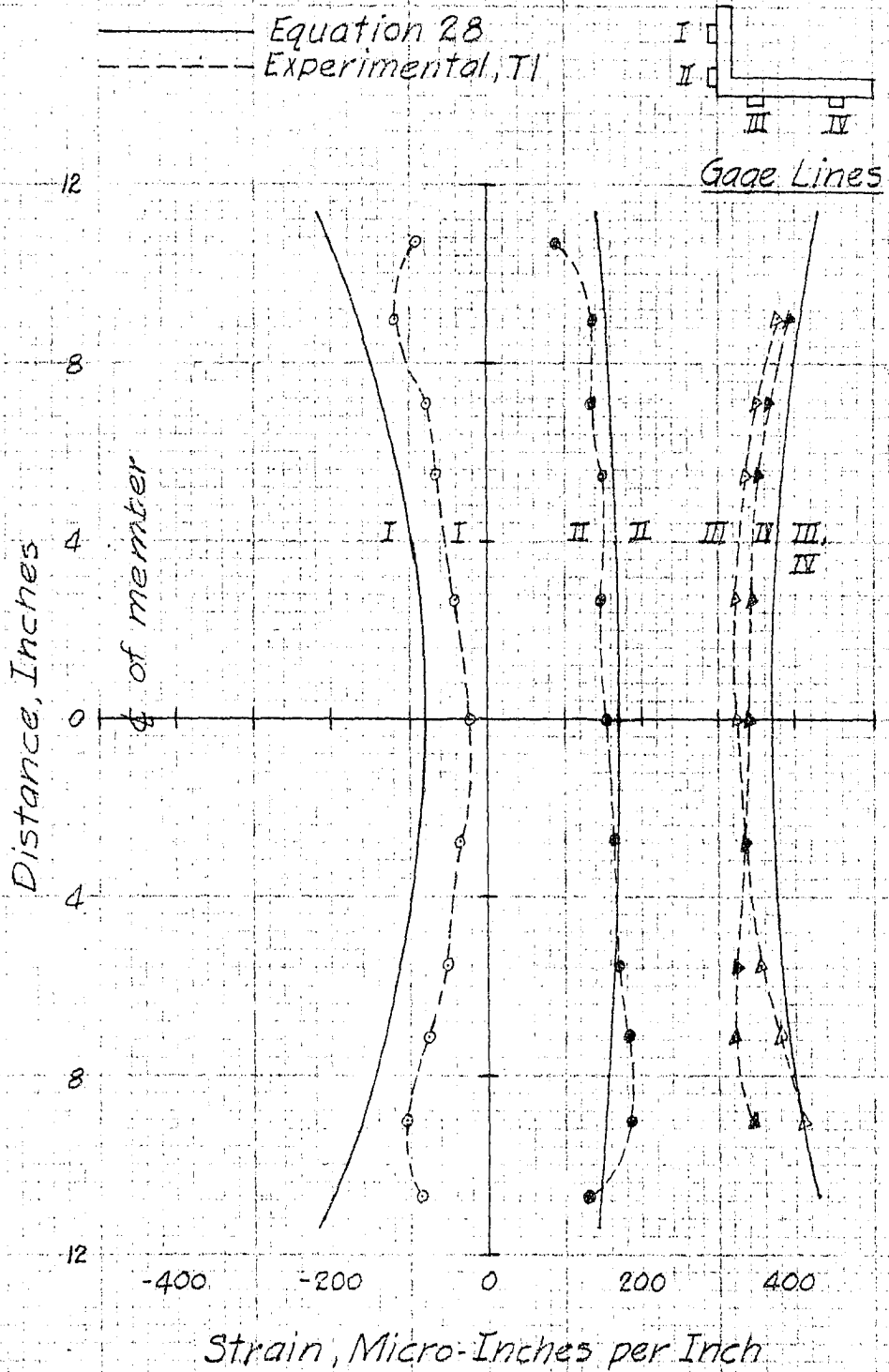
Figure 29. Member D4 in Tension, Comparison of Equation

(28) and Condition T1 for Load of 676 lb.

82
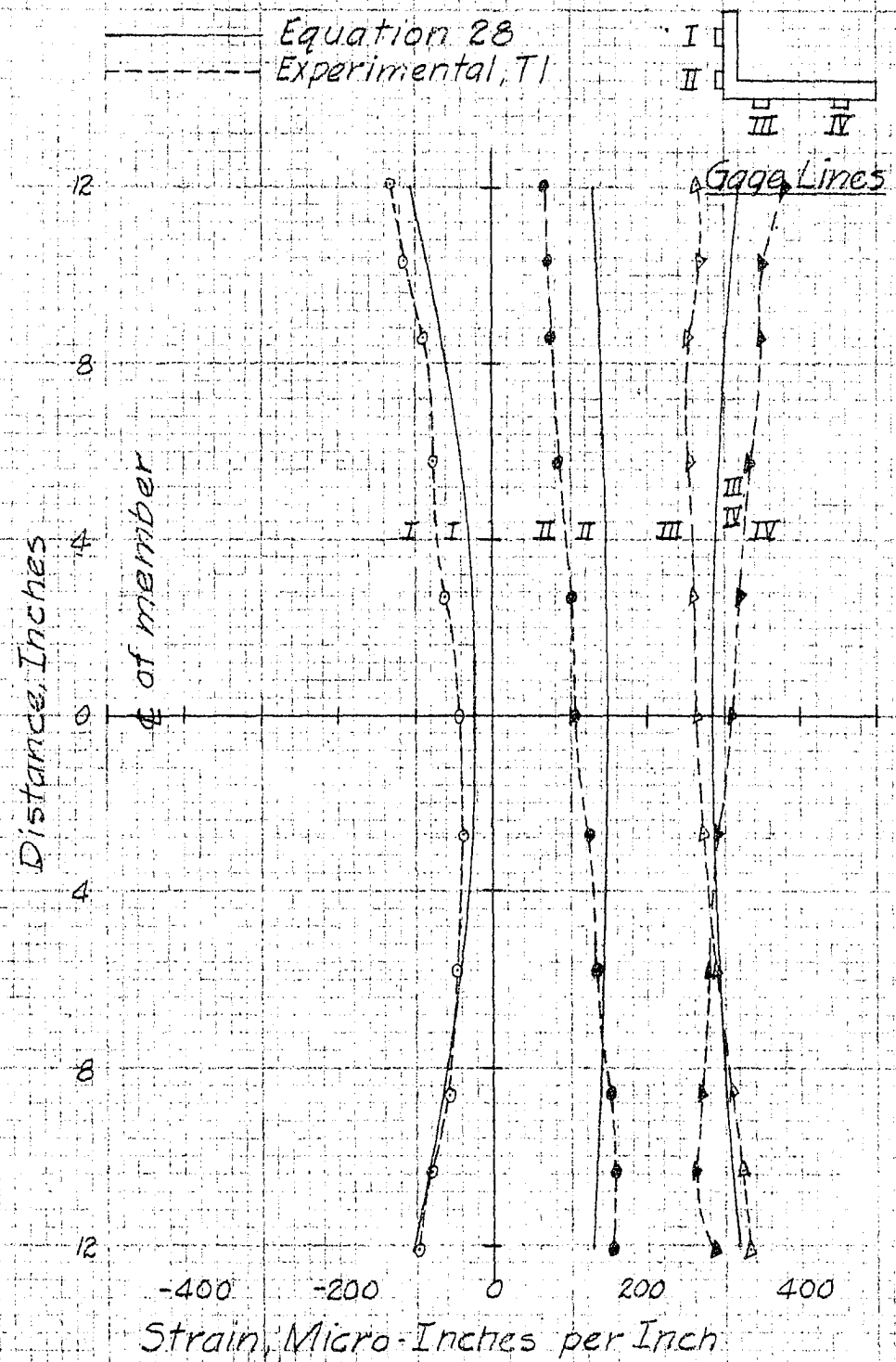
Figure 50. Member D5 in Tension, Comparison of Equation
(28) and Condition T1 for Load of 550 lb.
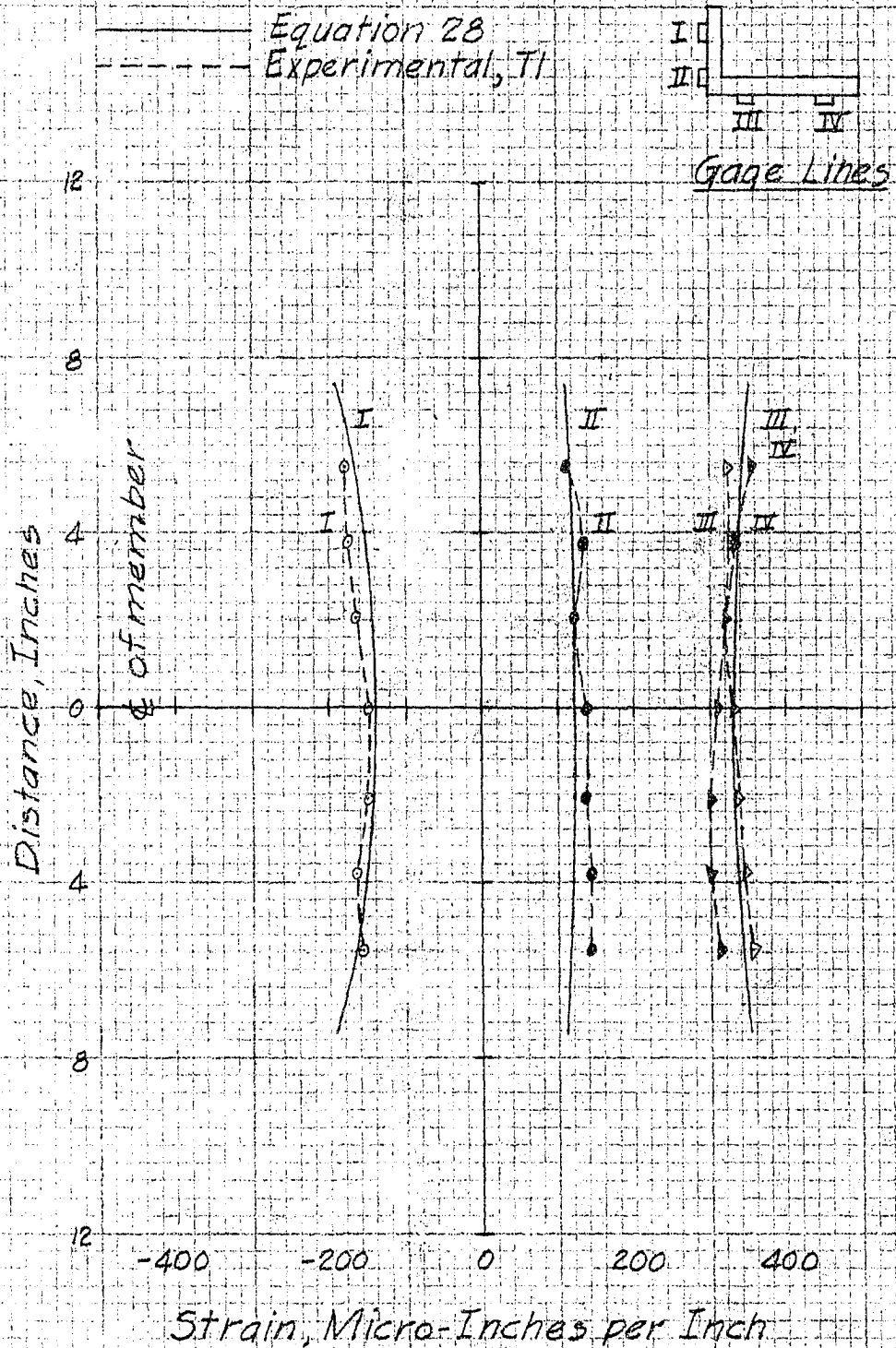
Figure 31. Member D7 in Tension, Comparison of Equation
(28) and Condition T1 for Load of 544 lb.

3. Effective length of members.

The tension member used in this investigation has two inflection points as indicated by the general shape of its elastic curve, shown in Figure 28. The length of the member between these inflection points is called the effective length. If this effective length can be established then the analysis of the tension member can be simplified. The member can be considered as a hinged end tension member of a length equal to this effective length.

At an inflection point the bending moment is zero and therefore from equation (19), at that point:

$$\frac{d^2y}{dx^2} = 0 \tag{29}$$

If the inflection point is between C and B in Figure 28 then x is between zero and ℓ_1 and equation (29) becomes:

$$\frac{d^2y_1}{dx^2} = 0 \tag{30}$$

Differentiating equation (26) twice and substituting in equation (30) gives:

$$\frac{d^2y_1}{dx^2} = \frac{M_c}{P} k_1^2 \cosh k_1 x = 0 \tag{31}$$

Since M_c and k_1 can be zero only when P is zero, and since $\cosh k_1 x$ cannot be less than one, this condition cannot exist. Therefore, the inflection point cannot be between B and C of the member.

In Figure 28 the section of the member from A to B is identical in form to the section from B to C, and similarly the inflection point cannot be at any point between A and B. Then, the inflection point must lie in the eccentric connection at B between the centroid of the angle and the center of the gusset plate. Therefore, the effective length of the tension member is the distance between the centroids of the bolted connections between the gusset plates and the angle member.

4. Development of practical method of design.

Equation (28), which was developed for tension members of the type tested in this investigation, is of value for theoretical investigations, determination of limits and development of practical methods of design. However, it is of very little direct value to the practical designer. Its use involves the same difficulties discussed for equation (10) for the compression member.

Therefore, it is highly desirable to develop a method of design which can be readily applied by the designer to his practical problems. Such a method will be developed on the basis of additional simplifying assumptions based on the theoretical and experimental results of this investigation.

The design of tension members subjected to direct and bending loads necessitates the use of a trial and error procedure where a member is selected and then checked to see whether it can safely support the design load. The maximum theoretical load for such a member is limited by a controlling unit stress. This controlling unit stress may be based on one of the following: a) the yield strength of the material; and, b) the localized failure where a relatively small element of a member fails due to compression or buckling. The

maximum theoretical load is reduced to determine the safe working load for the member. To accomplish this reduction a factor of safety is applied on the basis of prevailing conditions to be satisfied by the tension member.

As previously determined herein, the two inflection points for this member are located in the connections between the centroid of the angle and the centers of the gusset plates regardless of the magnitude of the load. Therefore, if these points could be definitely located the angle member could be considered as an eccentrically loaded hinged end tension member. The length of the member would be the effective length and the eccentricity would be the distance from the centroid of the angle to points of inflection.

The position of these points can be determined with equations (28) and (26). An examination of these equations, however, shows that the location is a function of the following: P, e, l_1, l_2, I_1, I_2, E_1 and E_2. Therefore, the inflection point is not fixed in position even for a particular member, but moves across the connection as the load varies.

On the basis of the foregoing discussion the maximum distance the inflection point can be from the centroid of the angle is the distance to the center of the gusset plates.

Then, it can be conservatively assumed that the inflection points are at the centers of the gusset plates, and the eccentricity at each end of the member is equal to the distance from the center of the gusset plate to the centroid of the angle.

For the tension member the maximum primary stresses will occur near the end of the member because the bending moment will be a maximum there. The bending moment will decrease toward a minimum at the center of the member because of the curvature of the member. Thus, the maximum stresses can be conservatively computed by the use of the familiar relationship $P/A \pm Mc/I$, where $M = Pe$ and e is the distance from the centroid of the angle to the center of the gusset plates.

A comparison of the computed strains, based on the foregoing assumptions, with the observed strains for the member D4 in tension in the machine, condition T1, and in the tower are shown in Figures 32 and 33. Similar curves for the members D5 and D7 appear in Figures A31 to A34 inclusive in Appendix A. In all cases the results of $P/A \pm Mc/I$ are conservative in comparison with the observed results. This reserve strength becomes available to compensate for the uncertainties in determining the magnitude of the loads and the effects of rolling and fabricating the members.
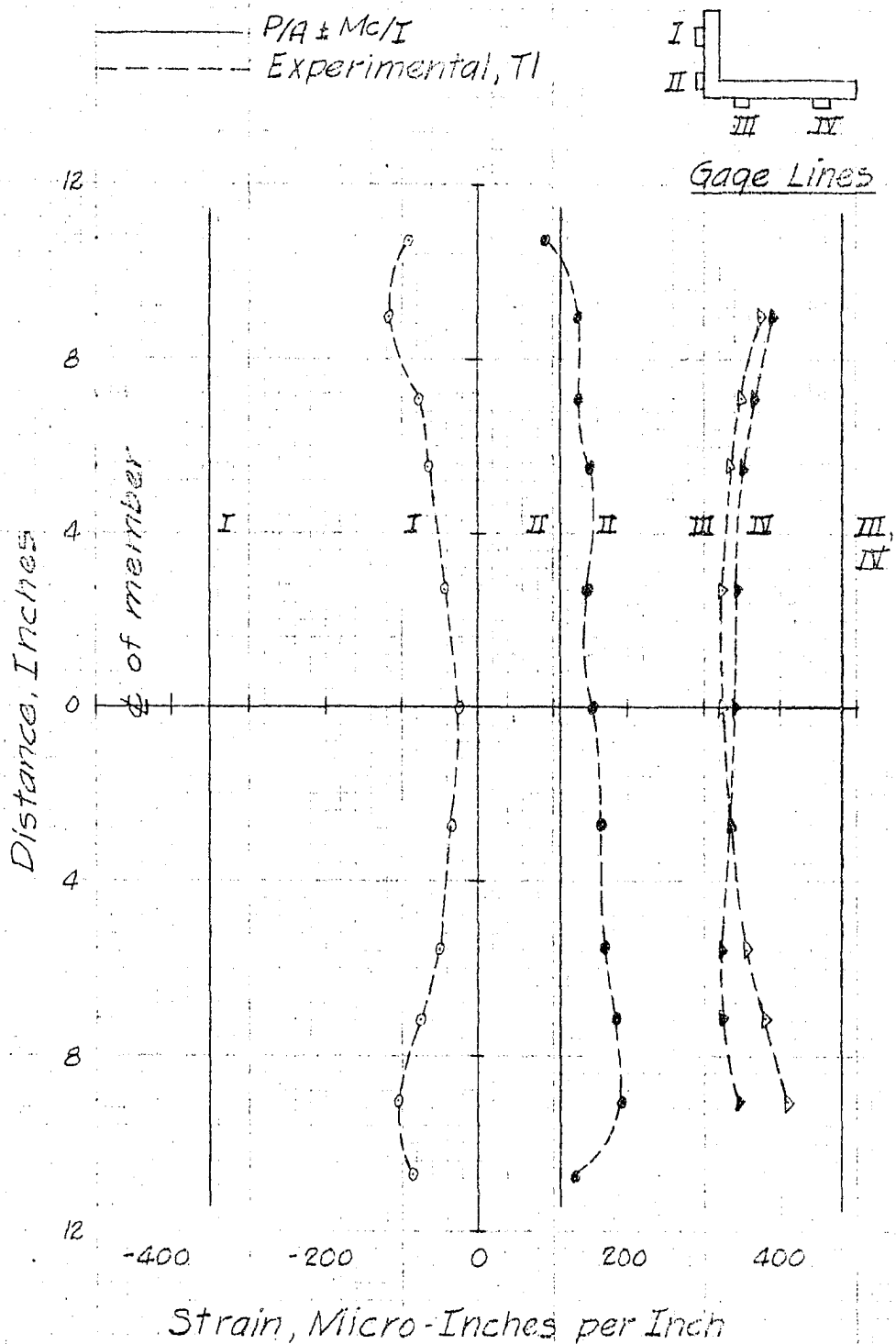
Figure 32. Member D4 in Tension, Comparison of P/A ± Mc/I

and Condition T1 for Load of 676 lb.
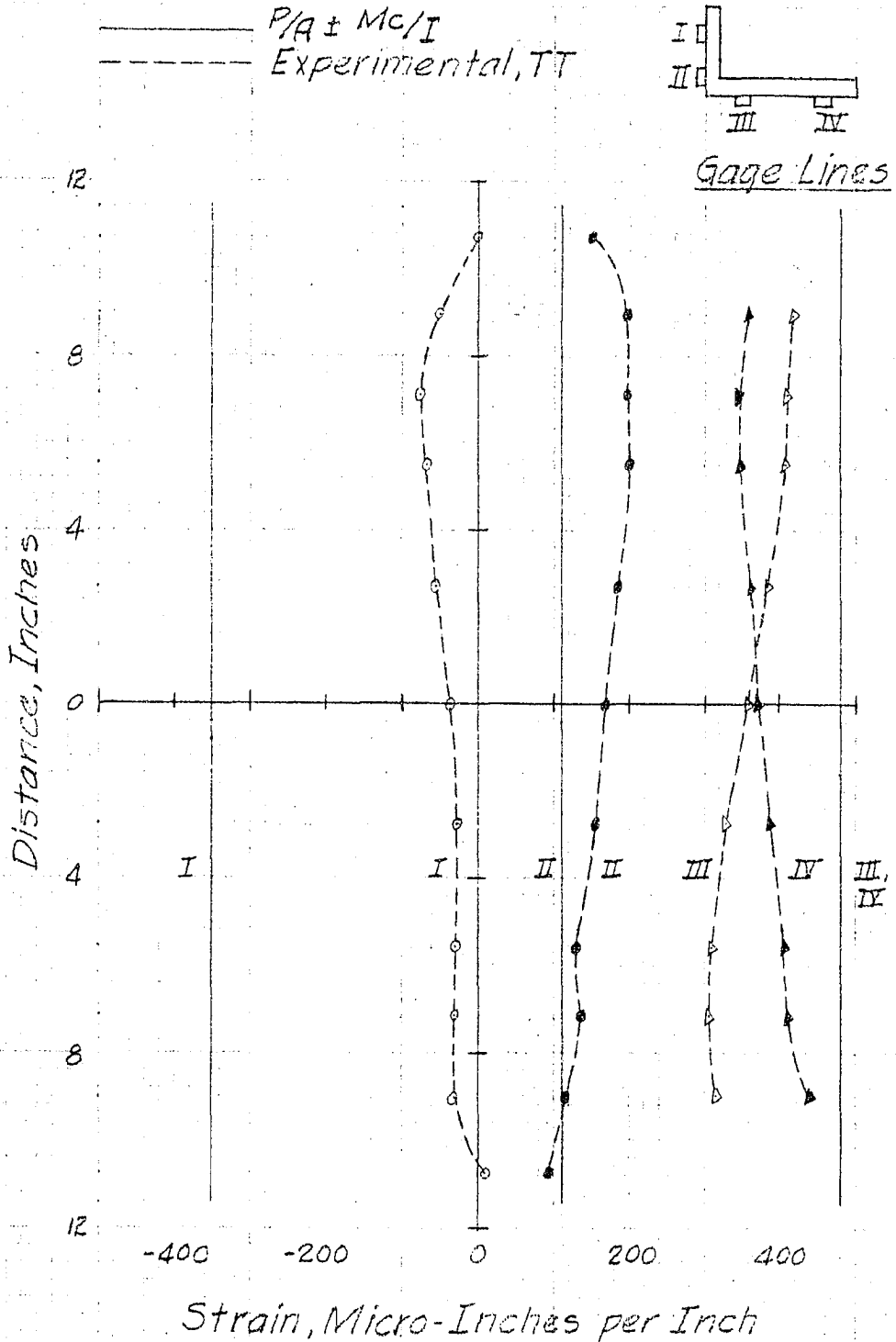
Figure 33. Member D4 in Tension, Comparison of P/A $\pm$ Mc/I and Condition TT for Load of 676 lb.

92

D. Evaluation of Results.

The value of the results of this investigation lies
in the information concerning the observed behavior of one
type of single-angle compression and tension members in an
aluminum space-frame tower. The three most important factors
of behavior of these members to the designer are the effec-
tive length, the direction of bending and the limits within
which the eccentricity varies. The observed results for
each of these factors and their applications in the design
of compression and tension members have been previously
discussed and compared with observed results. For com-
pression members this discussion is in III-B-4 and for
tension members in III-C-4 of this dissertation.

All of the results of this study were determined for
an aluminum structure, but since all of the stresses were
within the elastic range of the material the results can be
applied to any structural material within its elastic range.

The results of this investigation are based on the
observations of the behavior of unequal leg angle members
attached by the longer leg in a space-frame tower. There-
fore, these results are applicable only to this type of
member in this type of structure and additional investiga-
tions of other types of members and trussed structures are
necessary before any general application of such results
can be made.

1. <u>Compression members.</u>

A commonly accepted method of designing a single-angle compression member for a trussed structure is to consider the member as a concentrically loaded column. In this method the unsupported length of the member is assumed to be the distance between the working points (intersections of gage lines) of the member assuming no end restraint. In some cases this length is reduced by an arbitrary amount to compensate for the end restraint. The member is assumed to bend about the axis having the least radius of gyration and the eccentricities of the loads are ignored. The permissible load for the member is determined on the basis of a permissible average unit stress computed with a column formula for concentrically loaded members. Such a method is obviously quite arbitrary and based on several questionable assumptions as evidenced by the results of this study.

Nearly all design specifications provide for the use of some rational method for the analysis of eccentrically loaded compression members. The application of any of these methods to a single-angle member in an actual structure requires the designer to assume for the member the effective length, direction of bending and the magnitudes of the eccentricities. The effective length and the direction of bending are usually assumed as in the previous method, while

the eccentricities might be assumed as any one of several
distances. These distances might be from the centroid of
the angle to the center of the gusset plate or the back of
the angle, or any other distance the designer decides to use.
It is evident that in order to use one of these rational
methods the designer is required to make several decisions
and there is little available information on which he can
base these decisions. Therefore, he usually makes very
obviously conservative assumptions or he uses an arbitrary
method such as the one discussed previously in this section.

The results of this investigation furnish the informa-
tion necessary for the designer to make reasonable assump-
tions of the effective length, direction of bending and
magnitude of eccentricities for single-angle compression
members in a space-frame tower. With these reasonable
assumptions he can then use any one of the existing rational
methods of analyzing and designing these members such as the
secant formula as previously discussed in III-B-4.

For long flexible compression members as are commonly
used in tower type structures the members must also be
checked for general buckling of the member as a unit. Euler's
well known formula for critical load is used and again the
effective length and the direction of bending must be assumed.
The assumptions usually made for these factors are the same

as in the two previous methods of design. Again the results
of this investigation will assist the designer in making more
realistic assumptions of these factors as discussed previous-
ly in III-B-4.

2. Tension members.

The usual method of designing a single-angle tension
member for a trussed structure is to determine the permis-
sible load for the member. This permissible load is based
on the effective area of the angle and a specified permis-
sible average unit stress. The effective area is determined
differently according to various specifications. The AASHO
specifies the effective area as follows: (1, p. 169)

> The effective area of a single angle tension
> member, or of each angle of a double tension
> member in which the angles are connected back to
> back on the same side of a gusset plate, shall
> be assumed as the net area of the connected leg
> plus one-half of the area of the unconnected leg.

The net area of the connected leg is the gross area minus
the area of any bolt or rivet holes. The additional re-
duction in area of one-half of the unconnected leg is in-
tended to compensate for the eccentricity of the loads on
the member.

When designing under the AISC specification (2) the
effective area of an angle is the gross area of the angle
minus the area of any bolt or rivet holes in the attached

leg. No provision is made to compensate for the eccentricities of the loads.

This method of designing single-angle tension members in a trussed structure is very arbitrary. The results of this investigation provide the information necessary for the designer to make reasonable assumptions in regard to effective length, direction of bending and eccentricity, and to use a more rational method in the design of single-angle tension members in a space-frame tower. This type of design method and the necessary assumptions, based on the results of this study, have been previously discussed in III-C-4.

E. Suggestions for Future Study.

As a result of this investigation the following suggestions are made for future projects in this general area of trussed structures:

1. An extension of this project to include equal leg angles, unequal leg angles attached by the short leg and double-angles having various combinations of equal and unequal leg angles in various types of trussed structures.

2. An investigation of angle members continuous through panel points of various types of trussed structures.

3. An investigation similar to that recommended in items 1 and 2 but for other rolled sections.

4. An investigation similar to that recommended in items 1 and 2 but for built-up sections.

5. An experimental investigation of the elastic and inelastic buckling of members in trussed structures.

6. A detailed study of the strain distribution at the ends of members where they are connected to the gusset plates in a trussed structure.

7. A study of the behavior of members in trussed structures beyond the elastic range as related to limit design.

8. A study of the behavior of various shapes and arrangements of gusset plates in trussed structures.

IV. CONCLUSIONS

The following conclusions for primary elastic action
result from the theoretical and experimental investigations
presented herein and are considered applicable to the type
of single-angle members tested and the trussed space-frame
tower type of structure used in this investigation. All of
these conclusions are based on loads which do not produce
stresses beyond the elastic limit of the material. In
addition, for compression members the loads do not exceed
the critical loads for the angles or the gusset plates.

1. For the type of gusset plate used in this investigation
the end of the plate attached to the structure closely ap-
proaches the rigid-fixed condition for bending about the axes
in the plane of and perpendicular to the plane of the gusset
plate.

2. Practically all of the bending of a tension or com-
pression angle member throughout its length develops about
the axis parallel to the plane of the gusset plates.

3. The points of inflection at the two ends of a compression
or tension member, for bending about an axis parallel to the
plane of the gusset plates, are located:

   a. Laterally, between the limits of the axes of bend-
   ing of the angle and the gusset plates.

   b. Longitudinally, at the centroids of the bolt patterns
   at each end of the angle member.

4.   The effective length of a compression or tension member is the distance between the points of inflection at the two ends of the member as herein determined.

5.   When the effective length is established as stated in these conclusions, the rational formulas in present use for analyzing eccentrically loaded compression members yield results closely approximating those observed in this study.

6.   When the effective length is established as stated in these conclusions, Euler's well known formula can be used to compute the critical load for the angle member.

7.   When the limit of eccentricity, e, of the primary load is established as stated in these conclusions a tension member can be analyzed as an eccentrically loaded hinged-end member by using the expression $P/A \pm Mc/I$ where $M = Pe$.

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    Library. 1951.

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VII. APPENDICES

A. Appendix A: Additional Experimental Data.
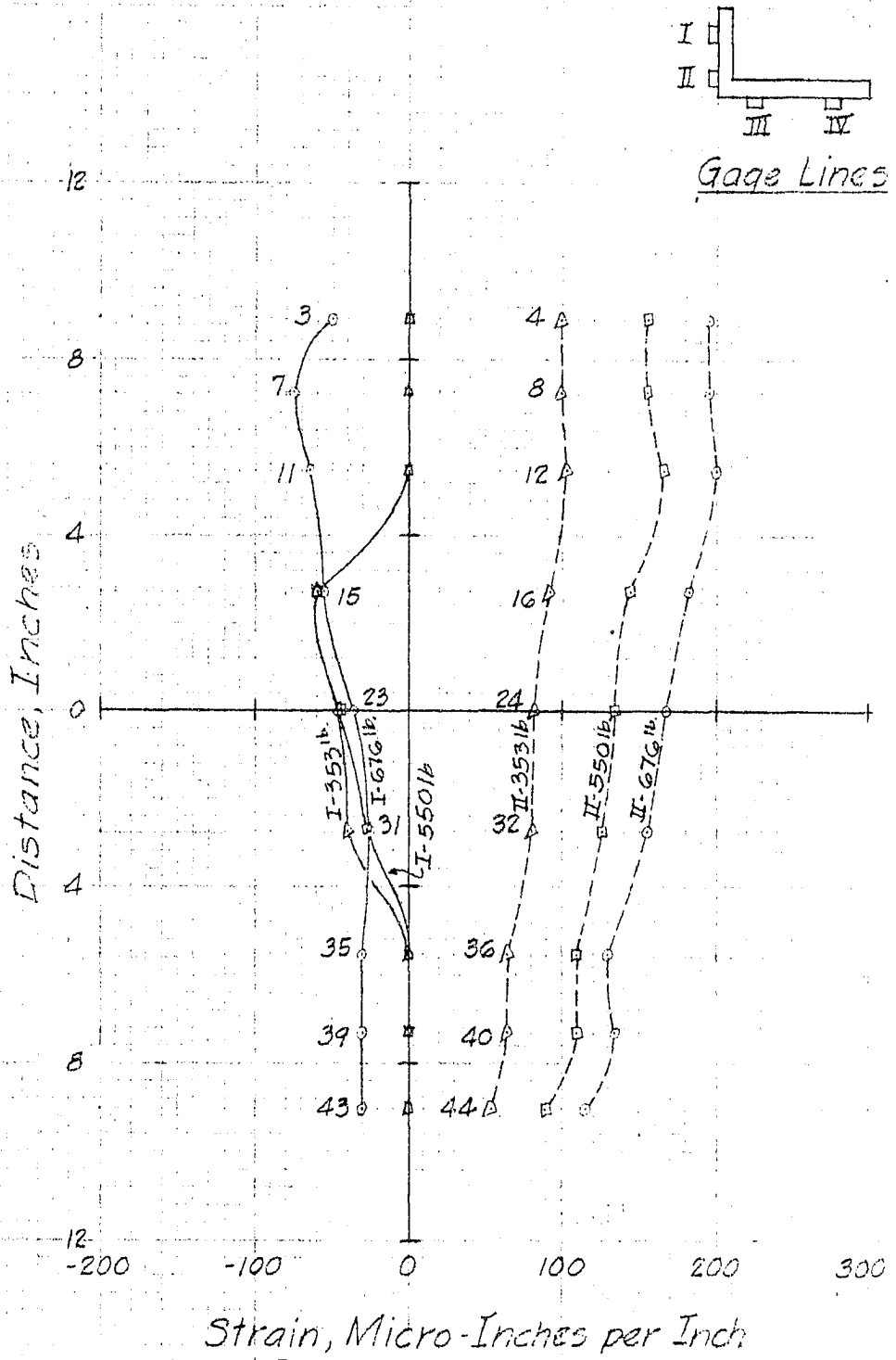
Figure A1. Member D40TT, Longitudinal Strain Curves for Gage Line I and II.
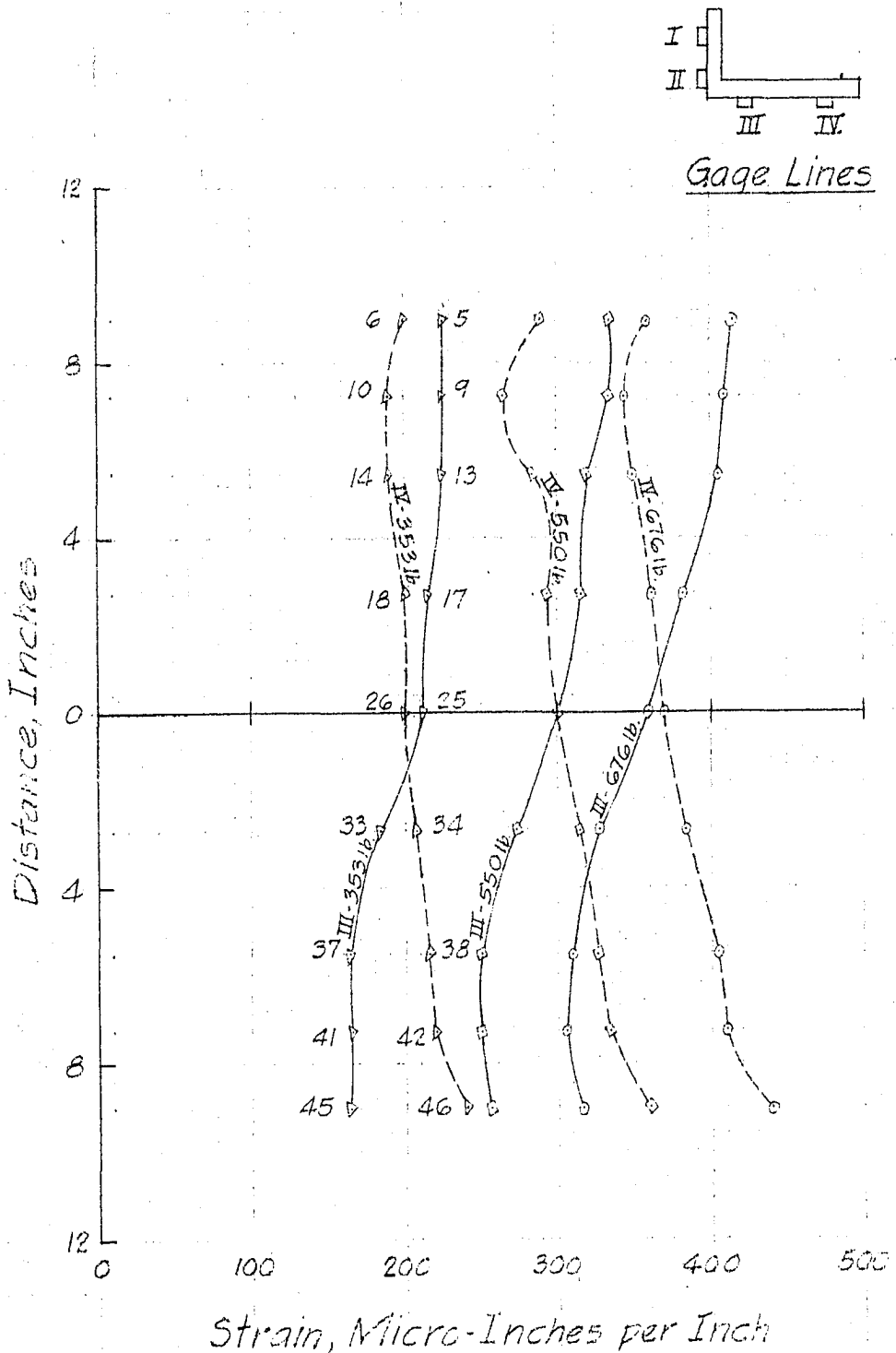
Figure A2. Member D4-TT, Longitudinal Strain Curves for
Gage Lines III and IV.
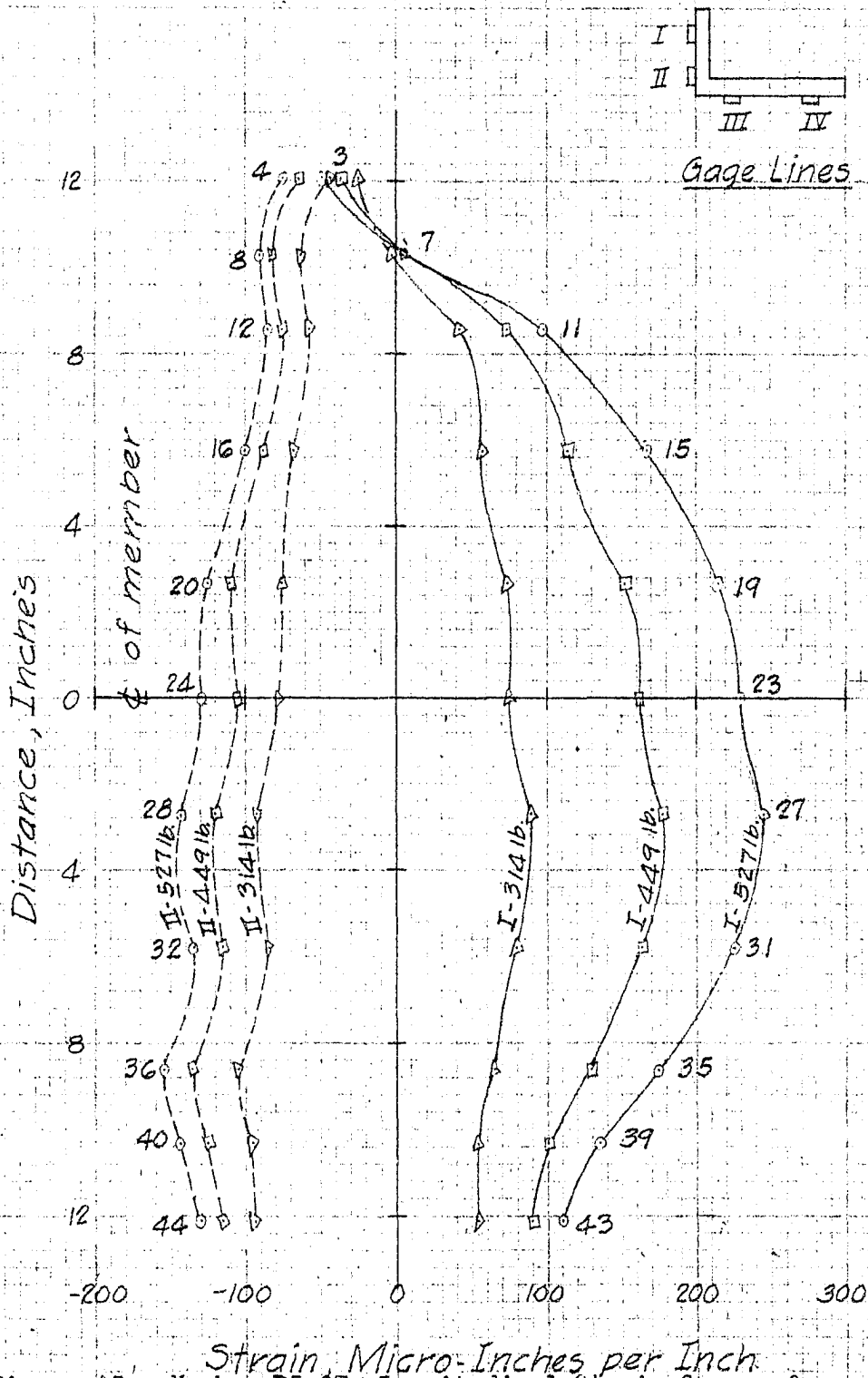
Figure A3. Member D5-CT, Longitudinal Strain Curves for
Gage Lines I and II.

107

Figure A4. Member D5-CT, Longitudinal Strain Curves for
Gage Lines III and IV.
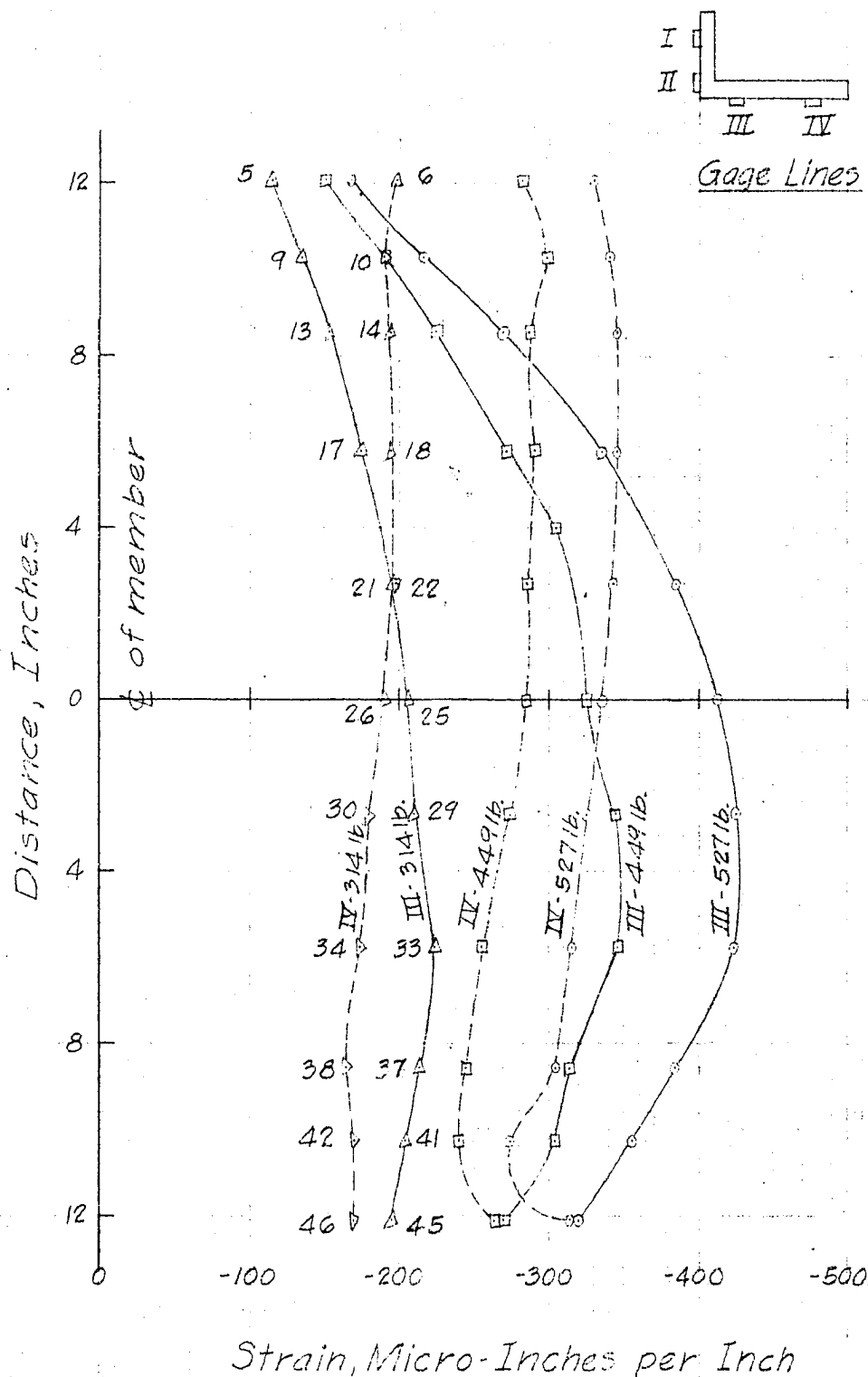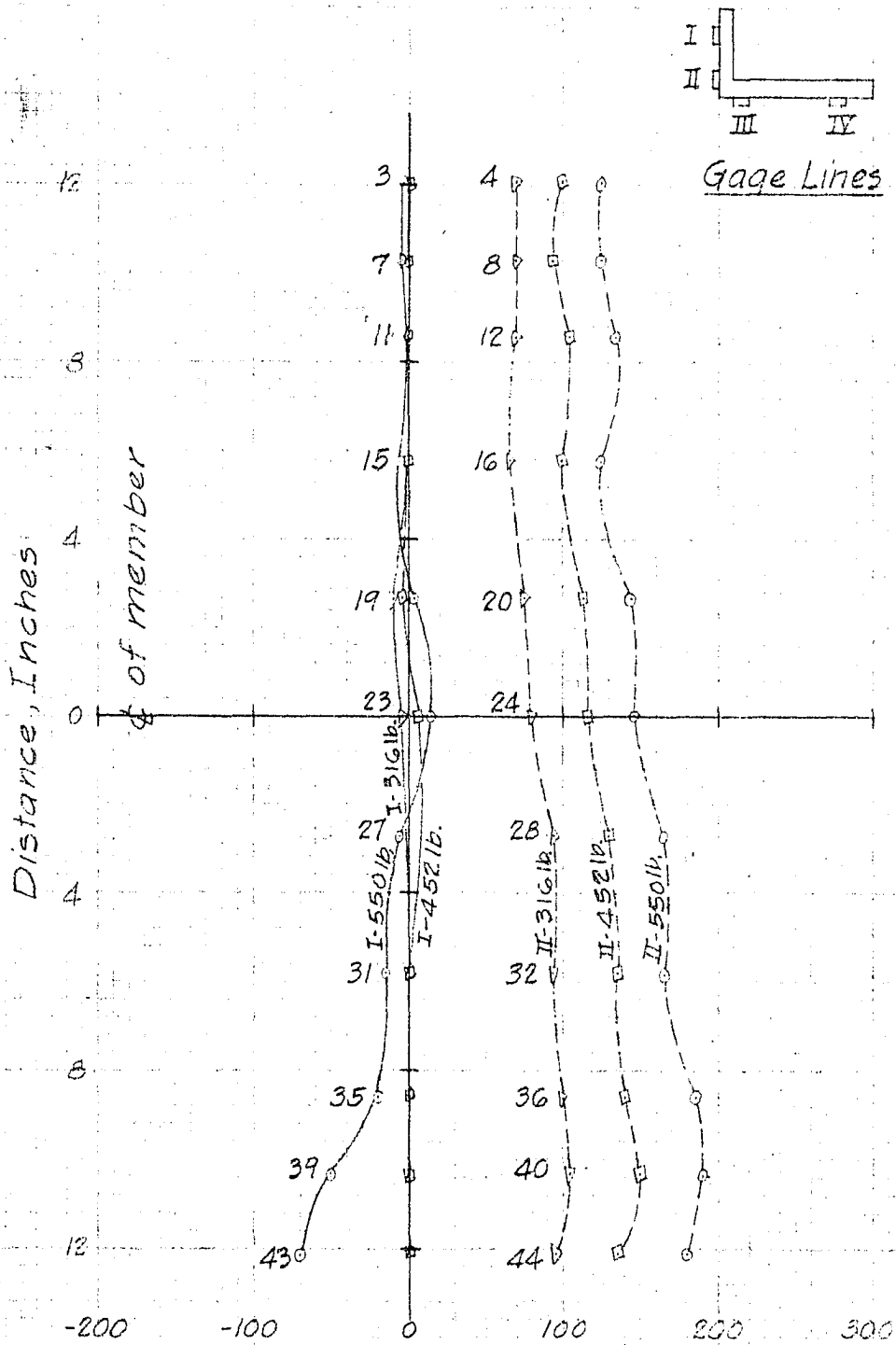
Figure A5. Member D5-TT, Longitudinal Strain Curves for Gage Lines I and II.

109
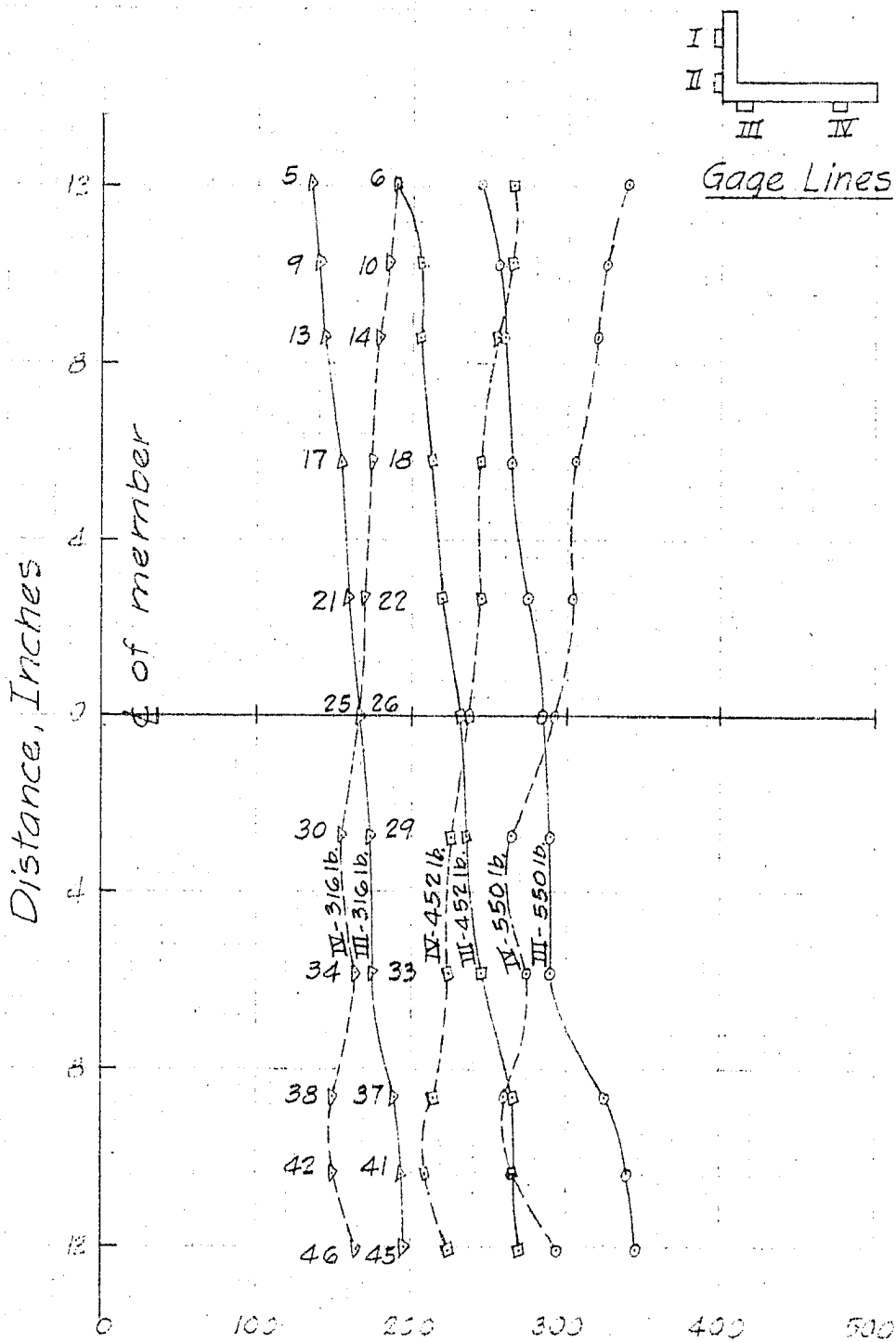
Figure A6. Member D5-TT, Longitudinal Strain Curves for Gage Lines III and IV.
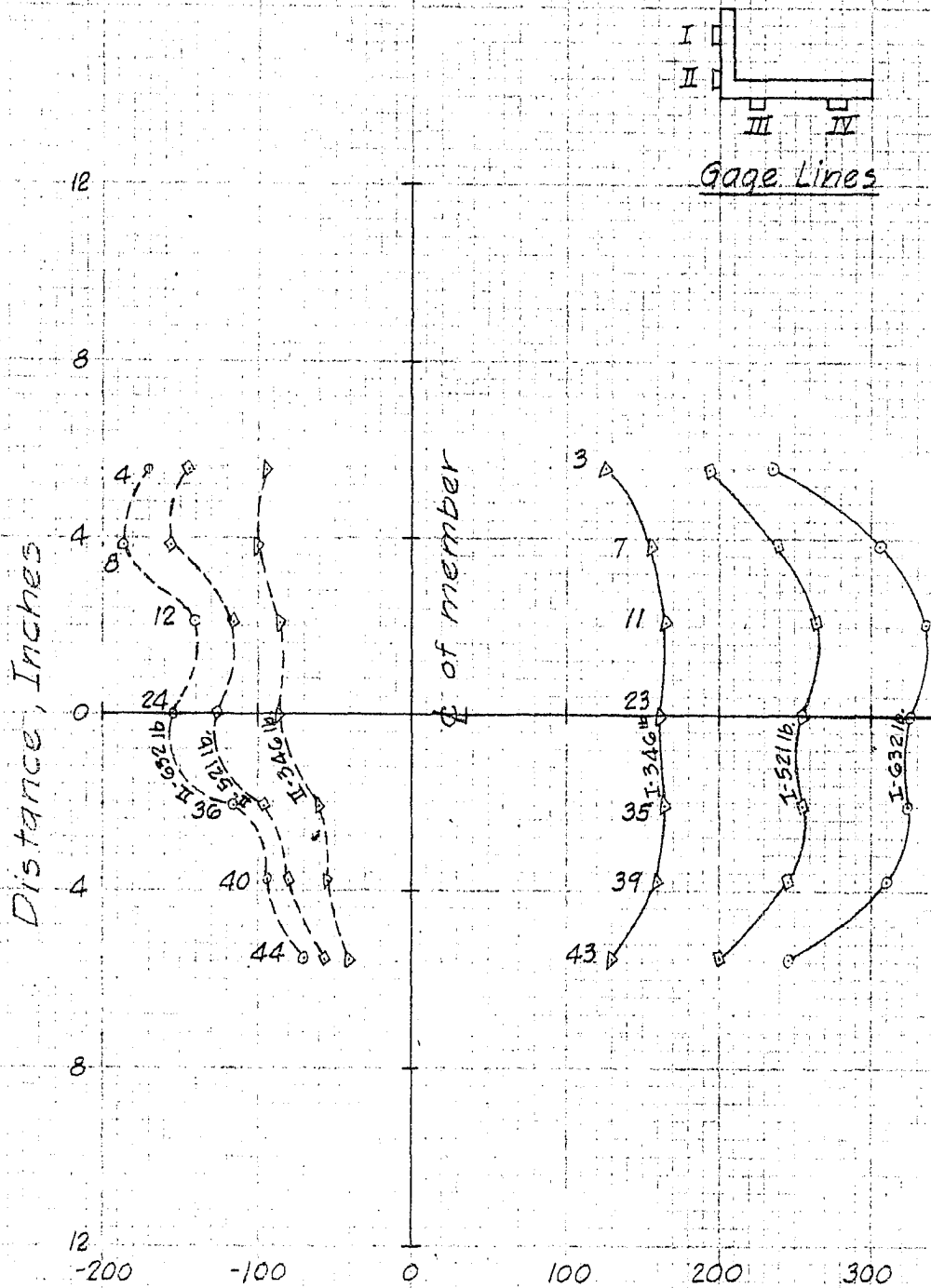
Gage Lines

Figure A7. Member D7-CT, Longitudinal Strain Curves

for Gage Lines I and II.

111
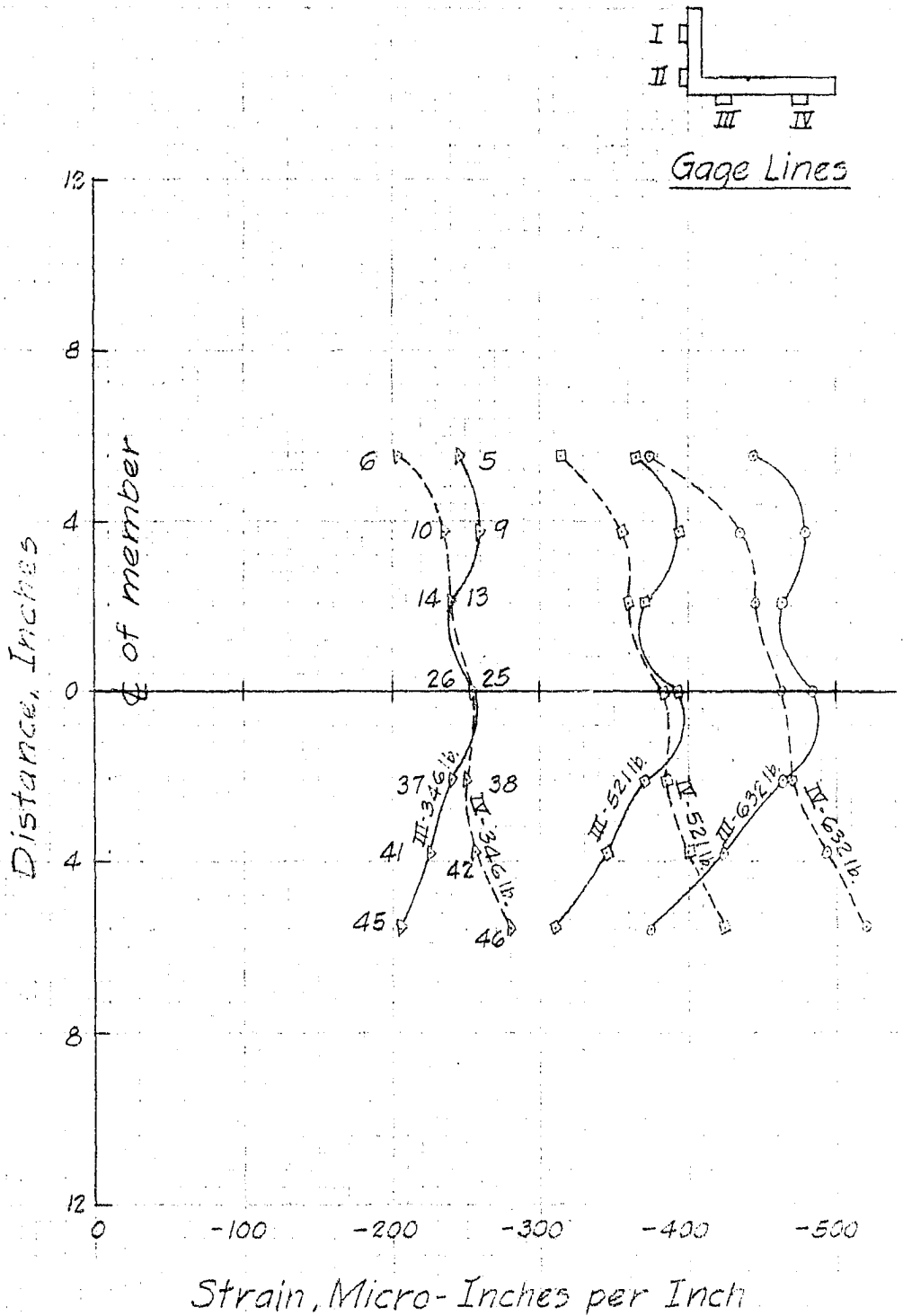
Figure A8.   Member D7-CT, Longitudinal Strain Curves

for Gage Lines III and IV.

112
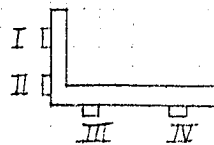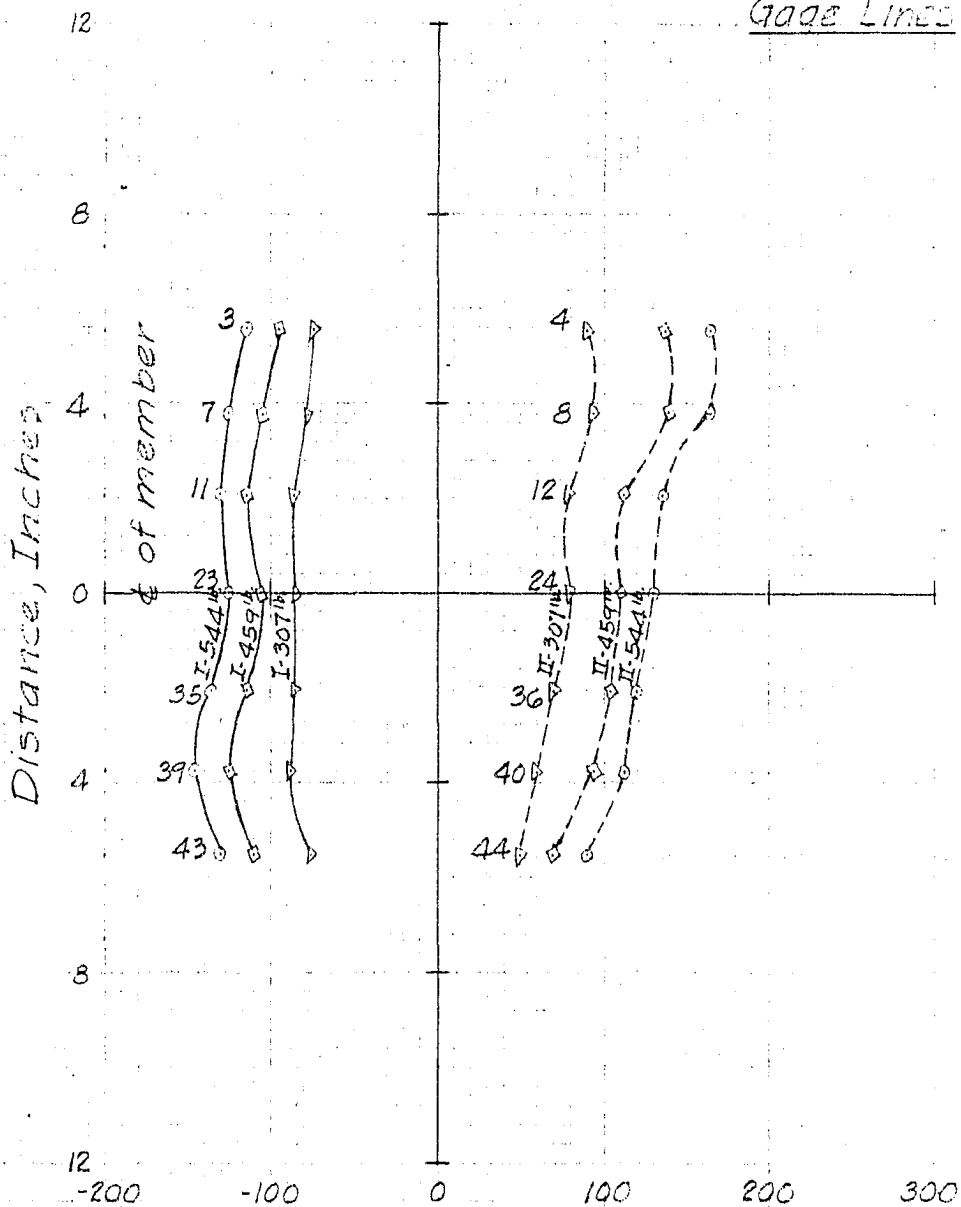
Figure A9. Member D7-TT, Longitudinal Strain Curves

for Gage Lines I and II.

113
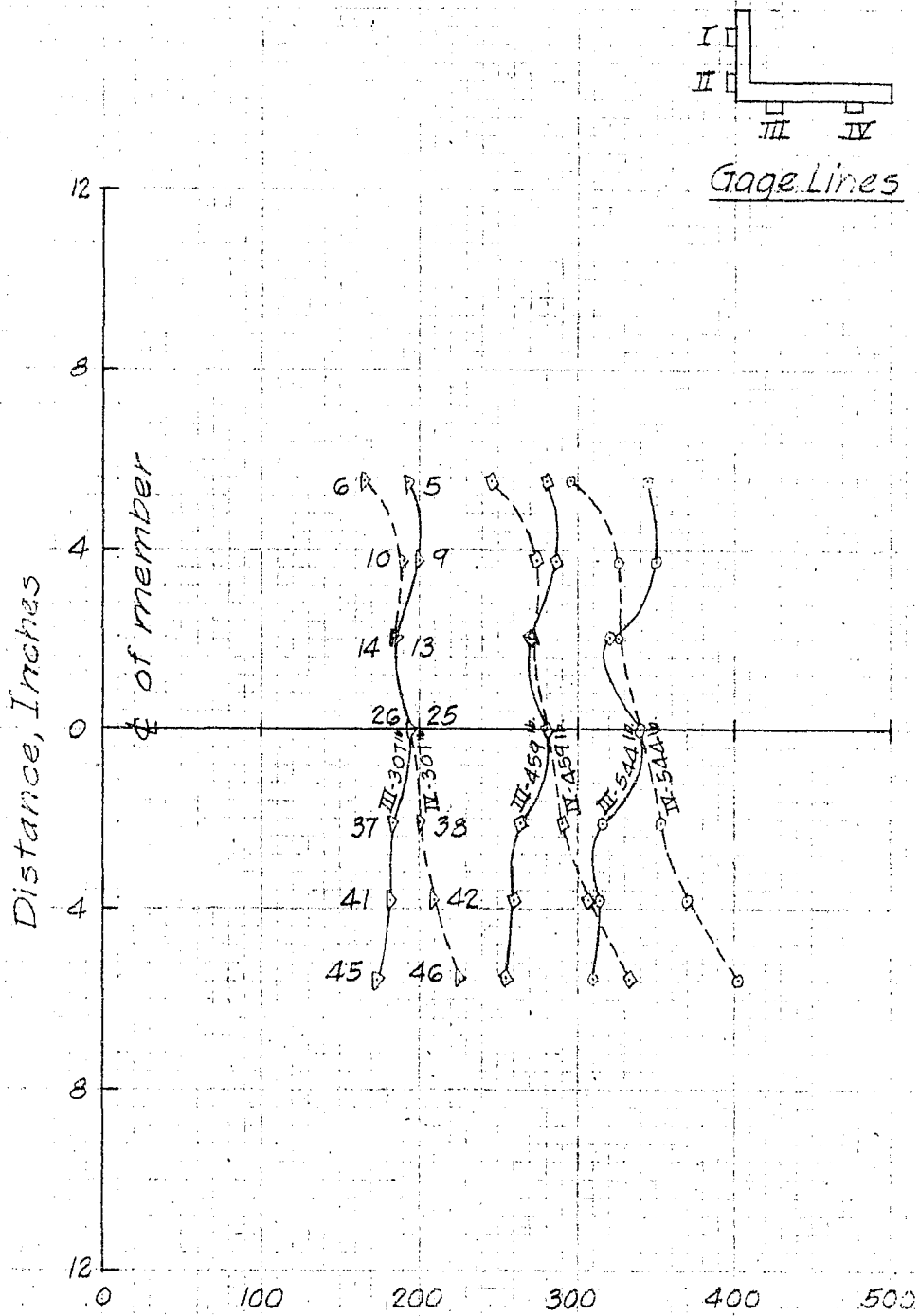
Figure A10. Member D7-TT, Longitudinal Strain Curves

for Gage Lines III and IV.
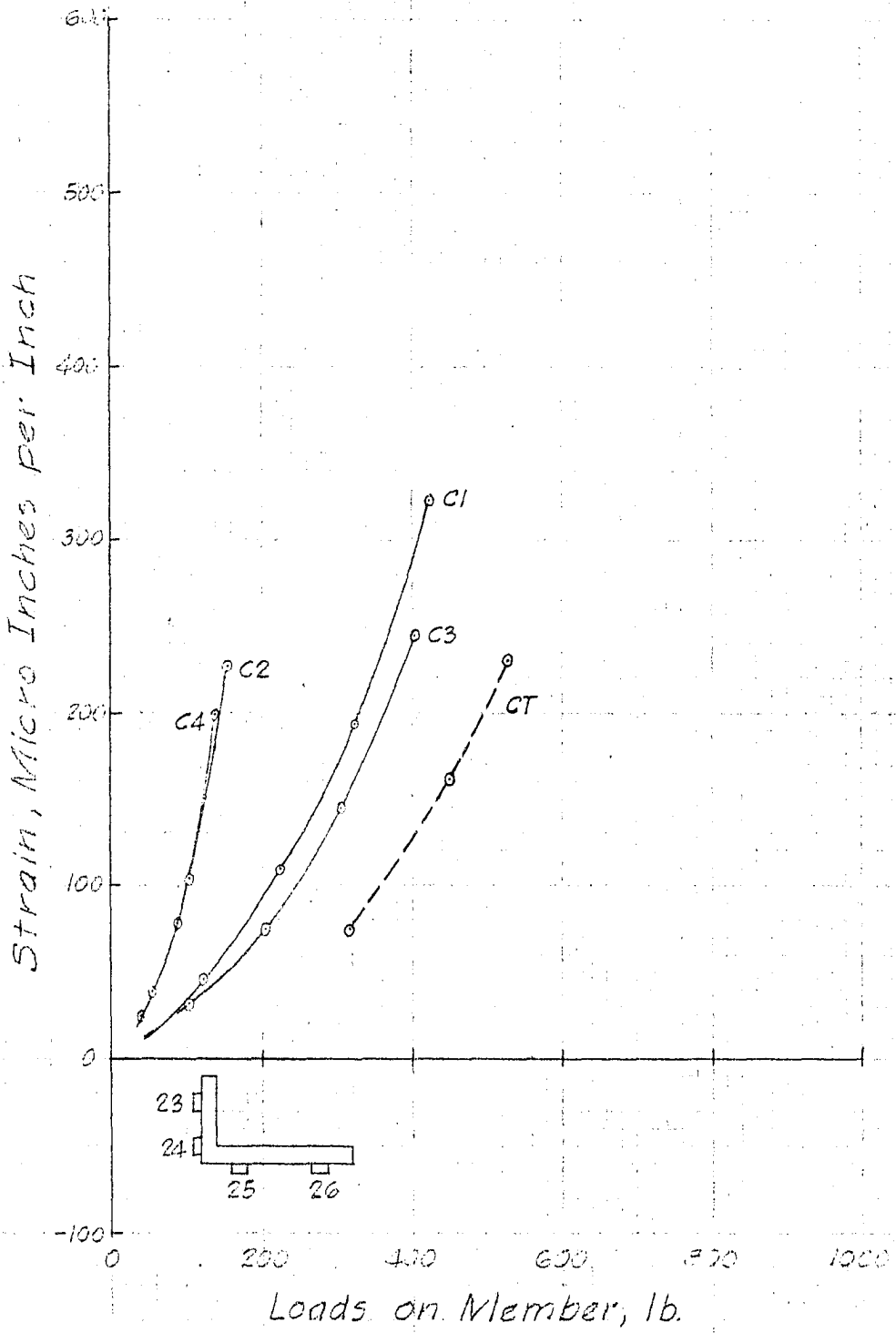
Figure A11.   Member D5 in Compression, Load-Strain Curves
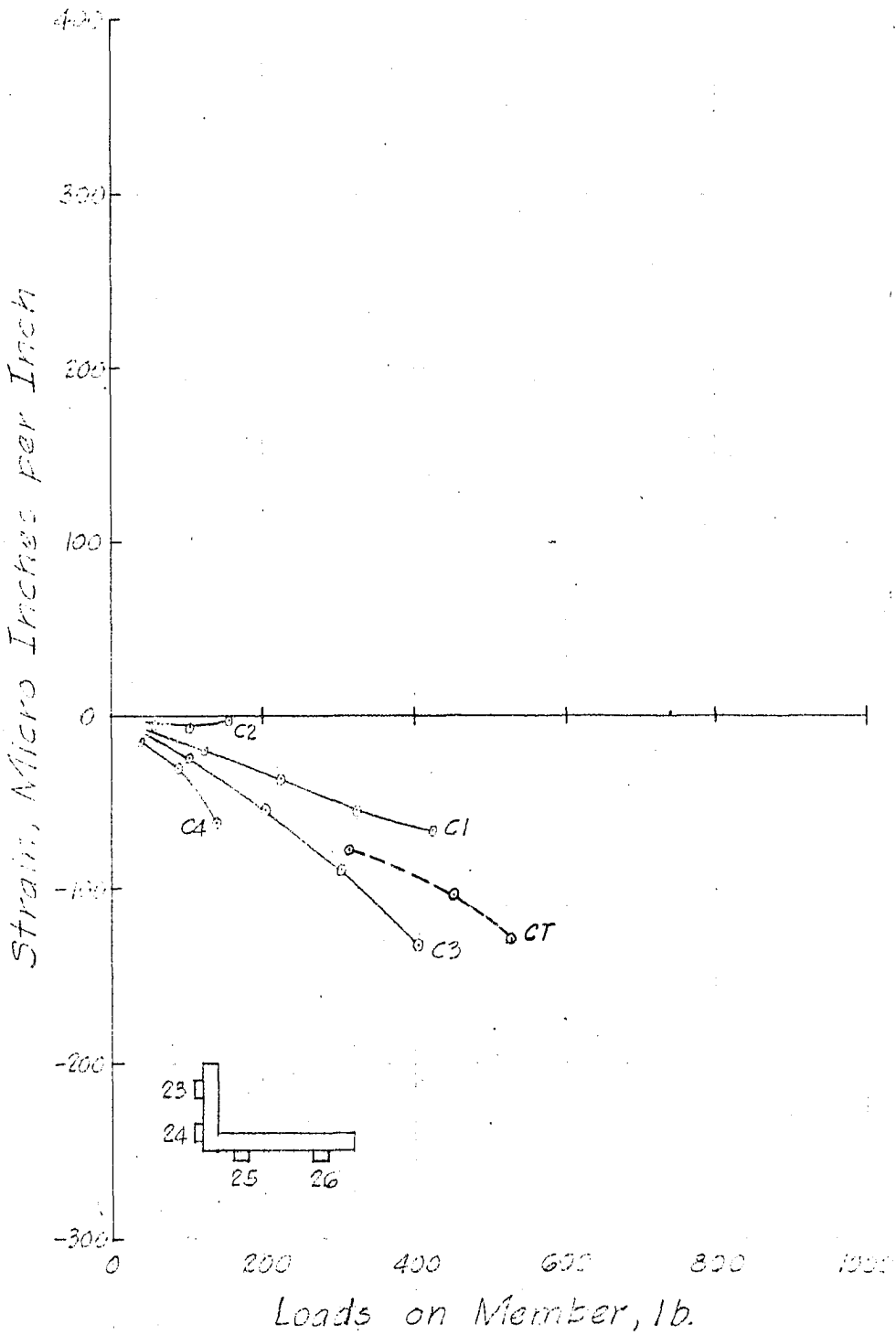
for Gage 23.

Figure A12. Member D5 in Compression, Load-Strain Curves
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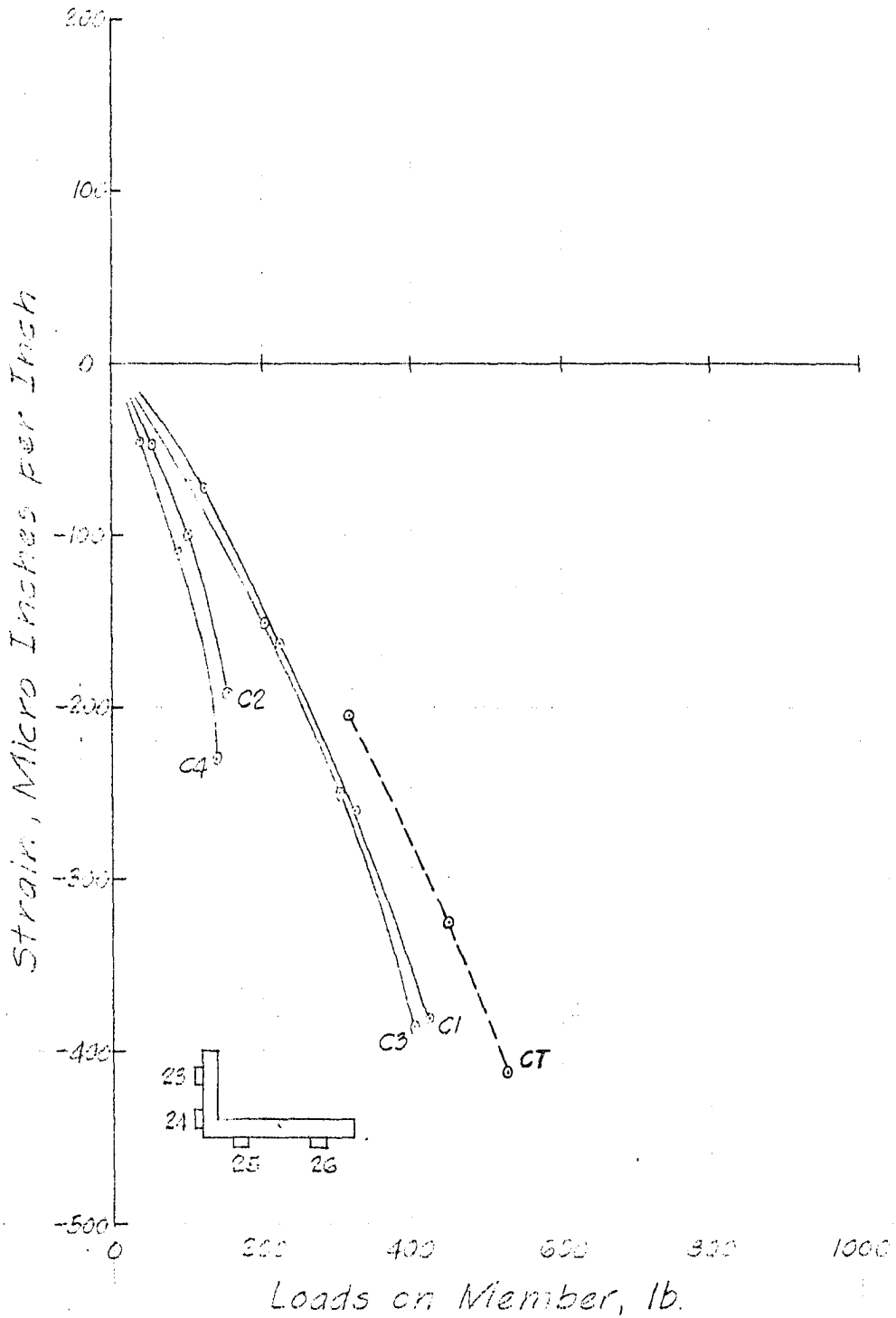
Figure A13. Member D5 in Compression, Load-Strain Curves
for Gage 25.
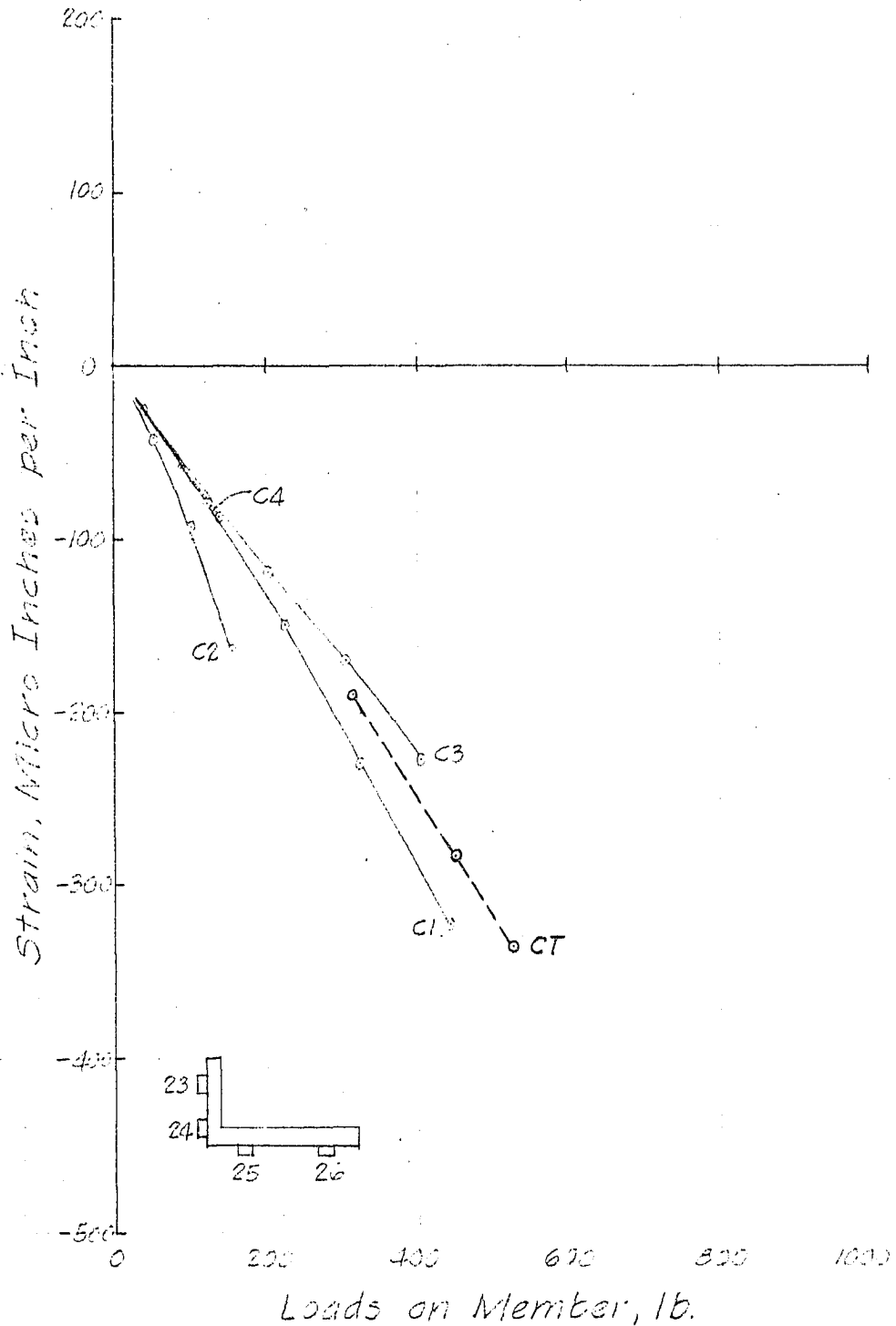
Figure A14. Member D5 in Compression, Load-Strain Curves
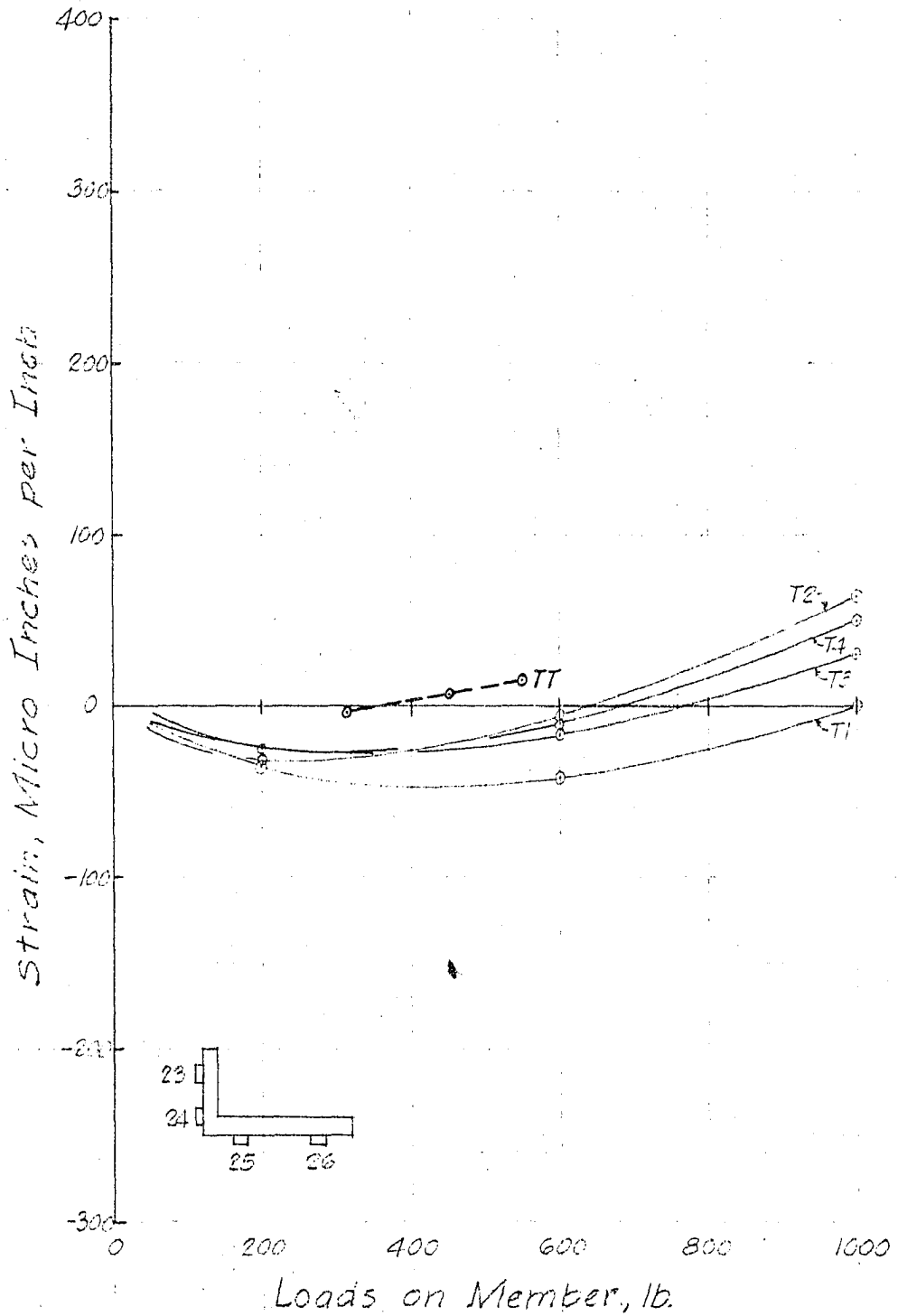
for Gage 26.

Figure A15. Member D5 in Tension, Load-Strain Curves
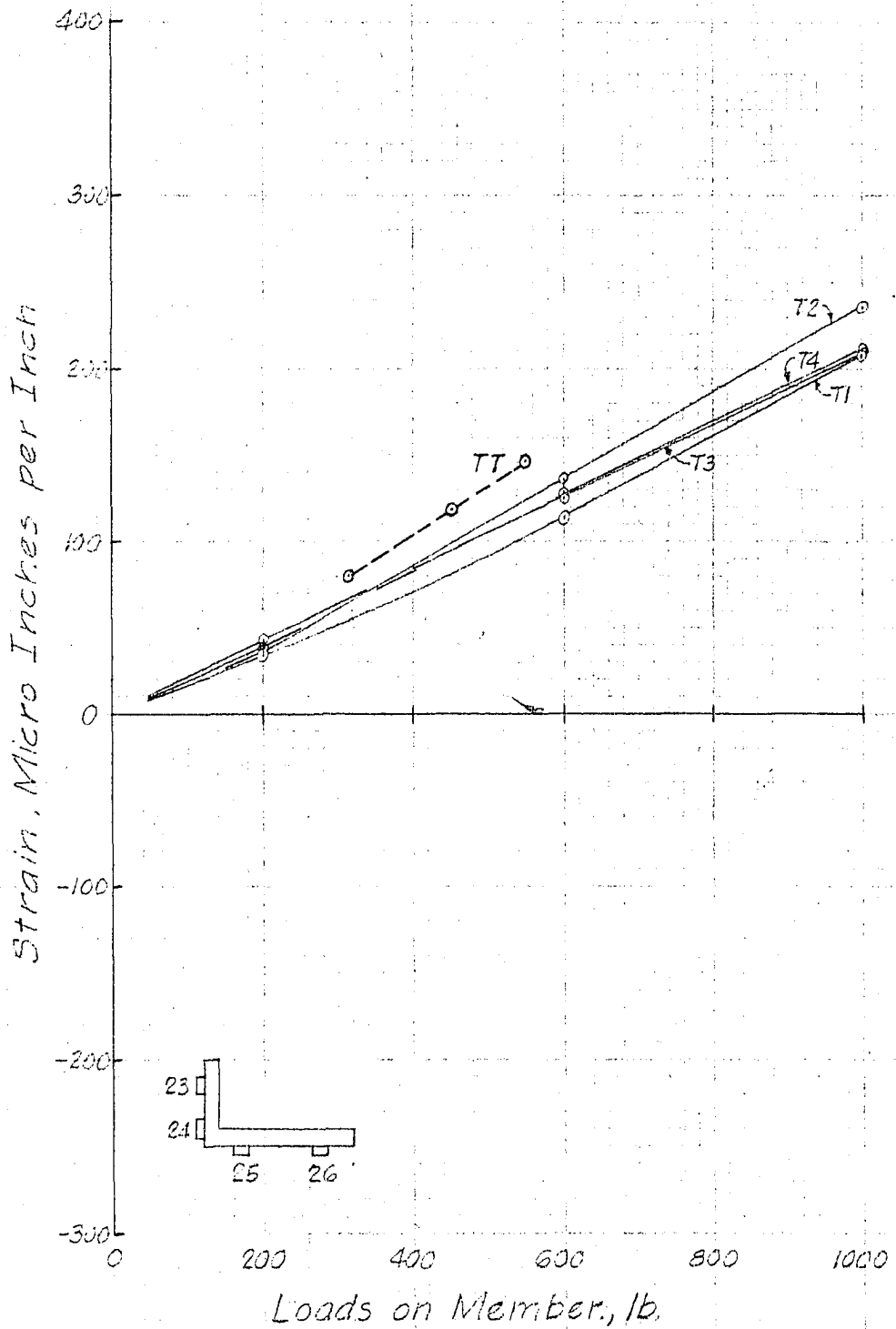
for Gage 23.

Figure A16. Member D5 in Tension, Load-Strain Curves
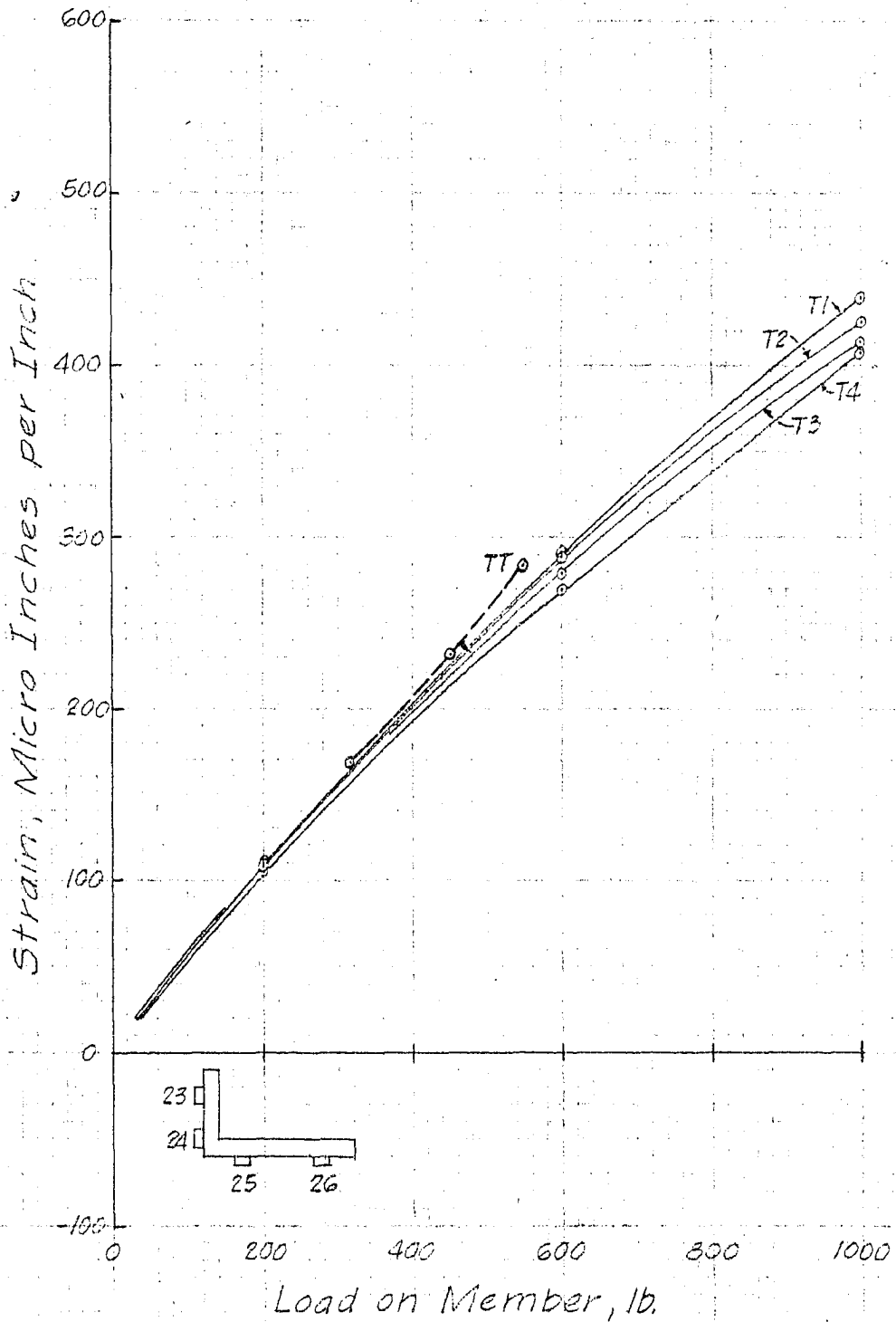
for Gage 24.

Figure A17. Member D5 in Tension, Load-Strain Curves
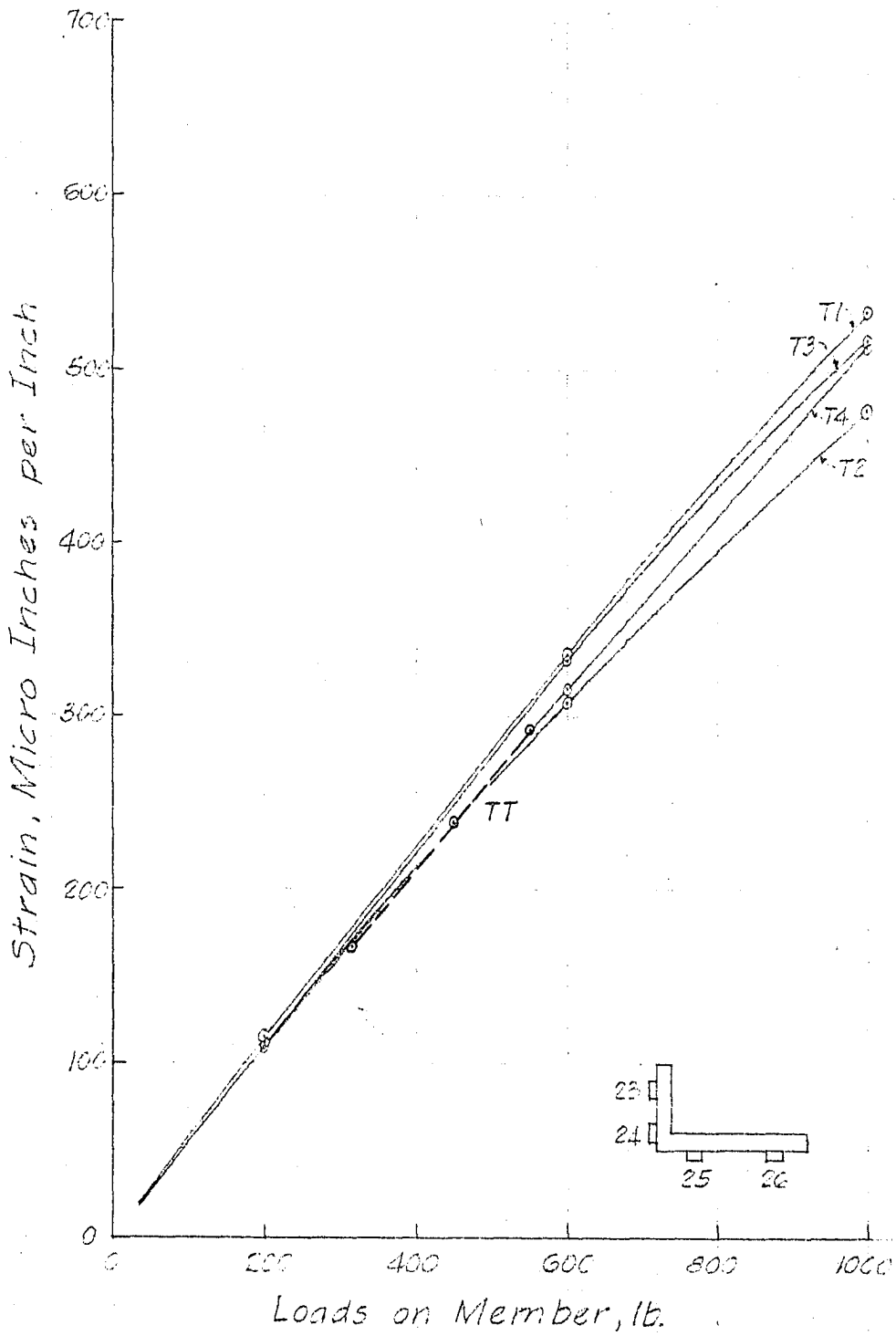
for Gage 25.

Figure A18. Member D5 in Tension, Load-Strain Curves
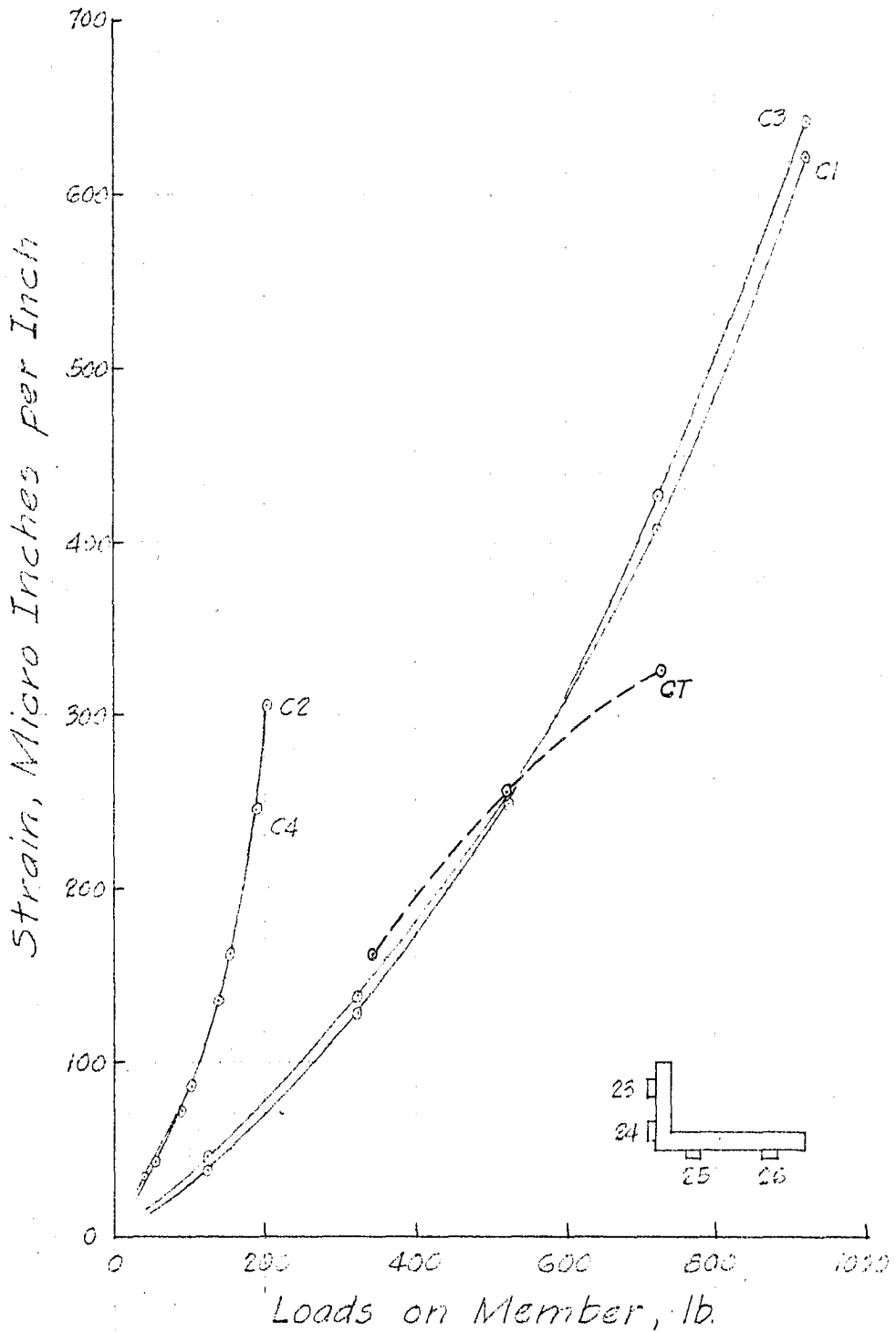
for Gage 26.

Figure A19.   Member D7 in Compression, Load-Strain Curves
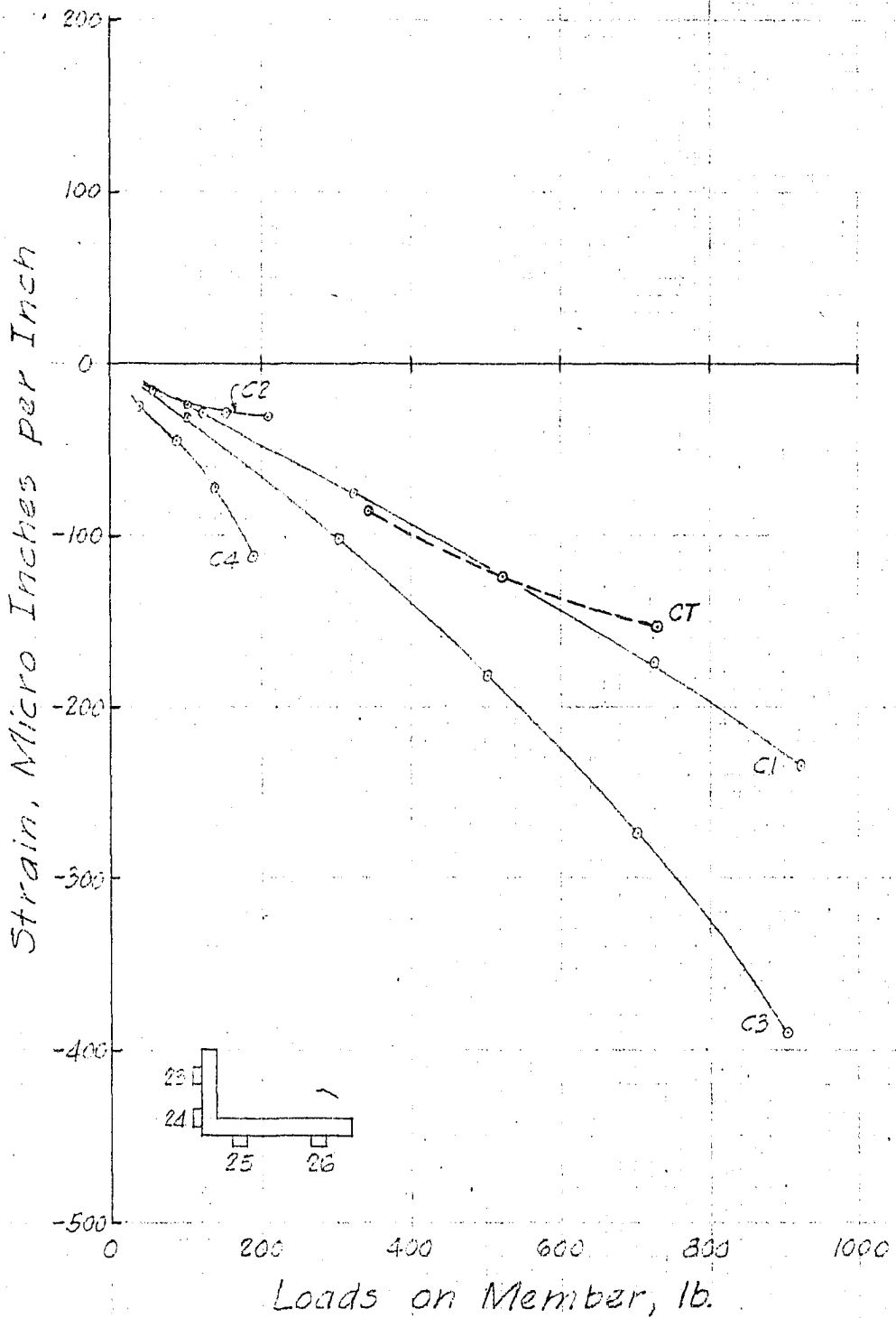
for Gage 23.

Figure A 20.   Member D7 in Compression, Load-Strain Curves
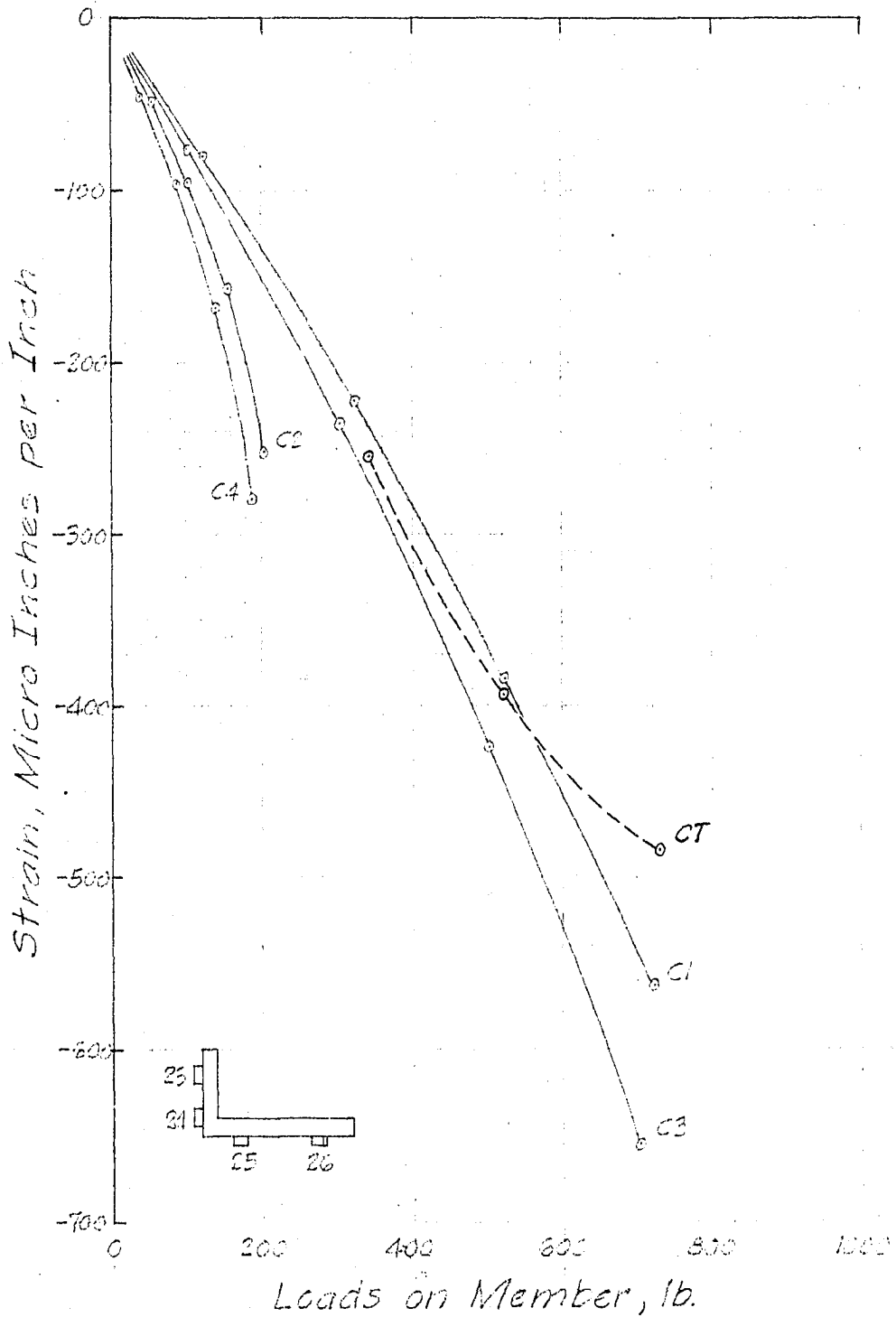
for Gage 24.

Figure A21. Member D7 in Compression, Load-Strain Curves
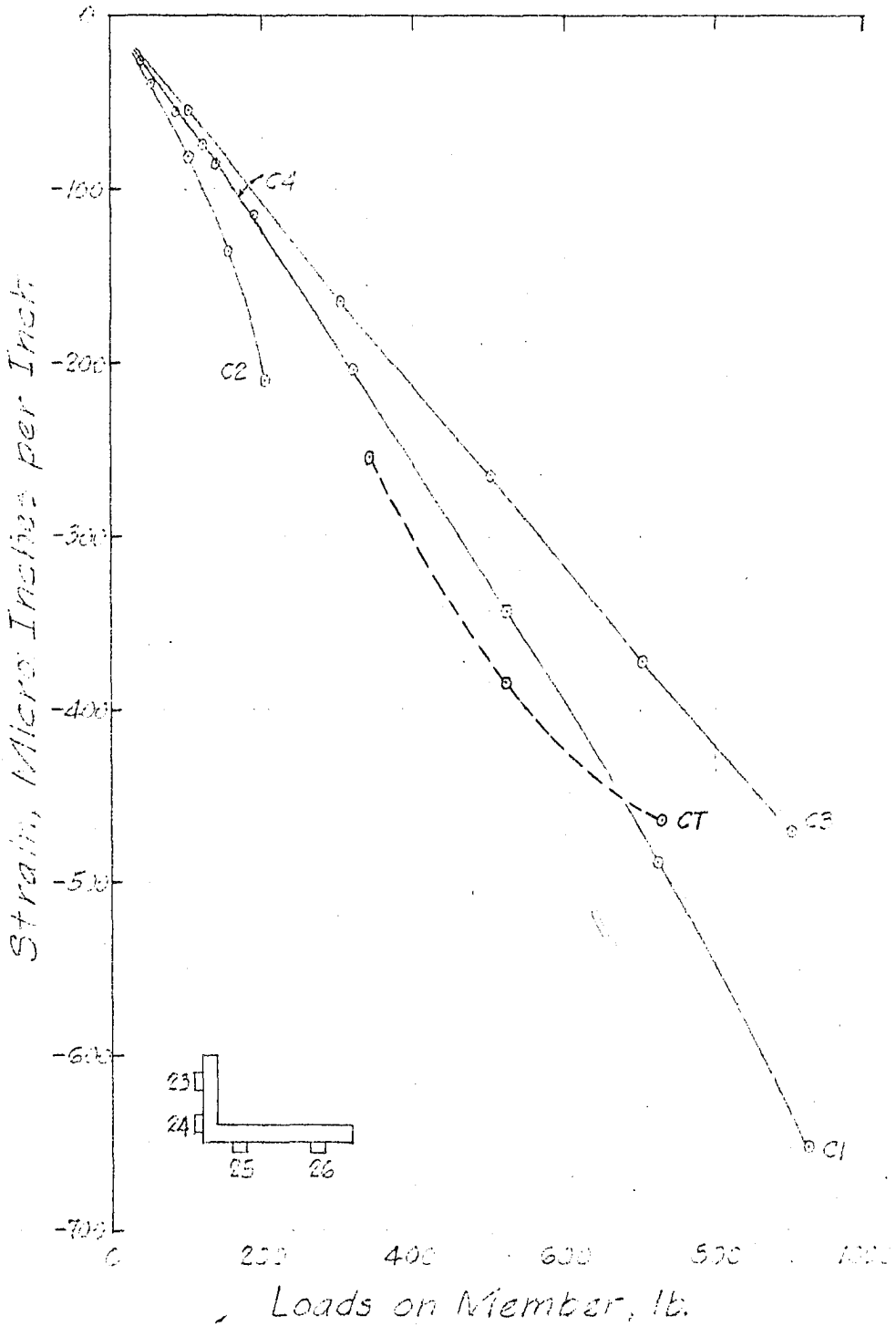
for Gage 25.

Figure A22. Member D7 in Compression, Load-Strain Curves
for Gage 26.
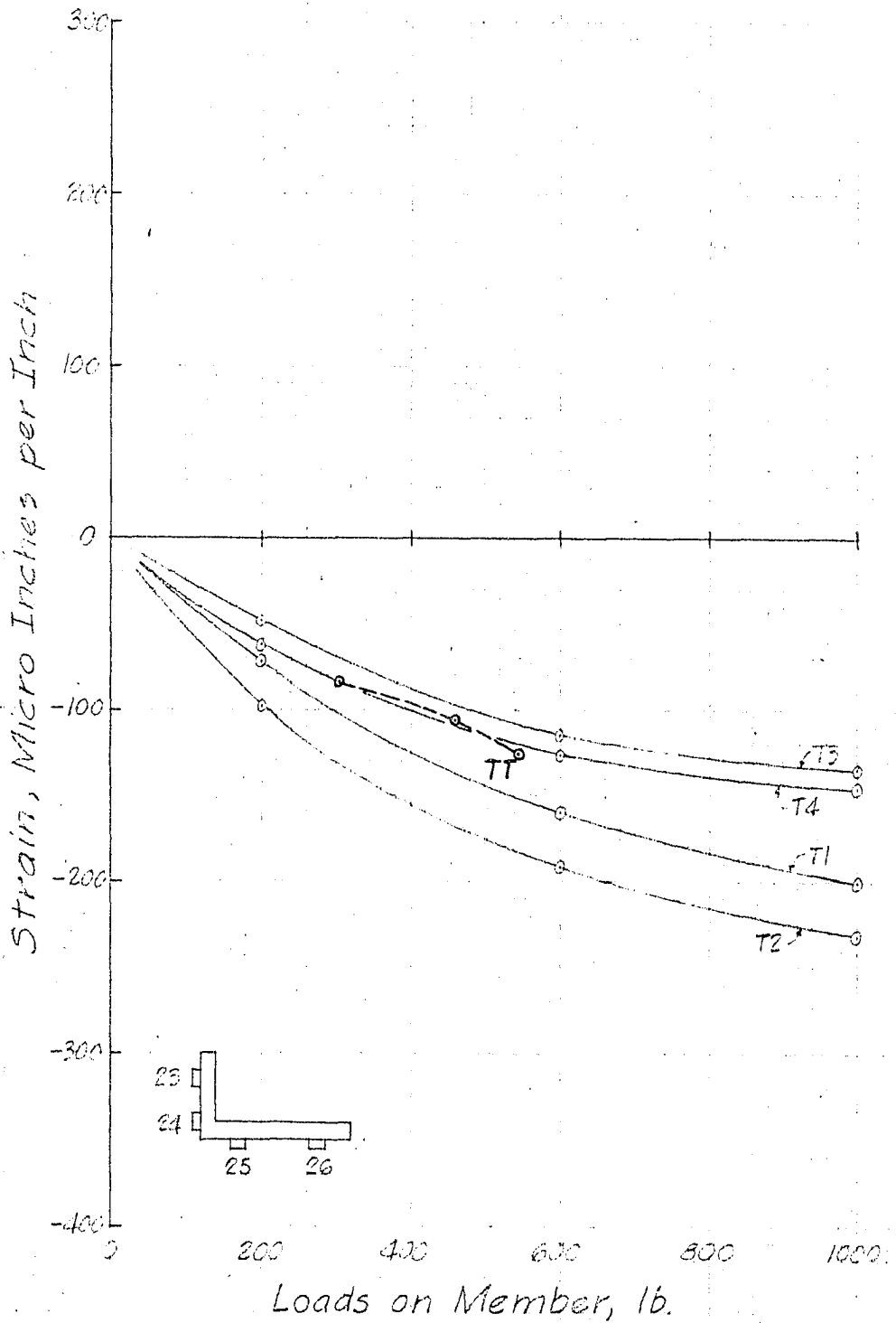
Figure A23. Member D7 in Tension, Load-Strain Curves
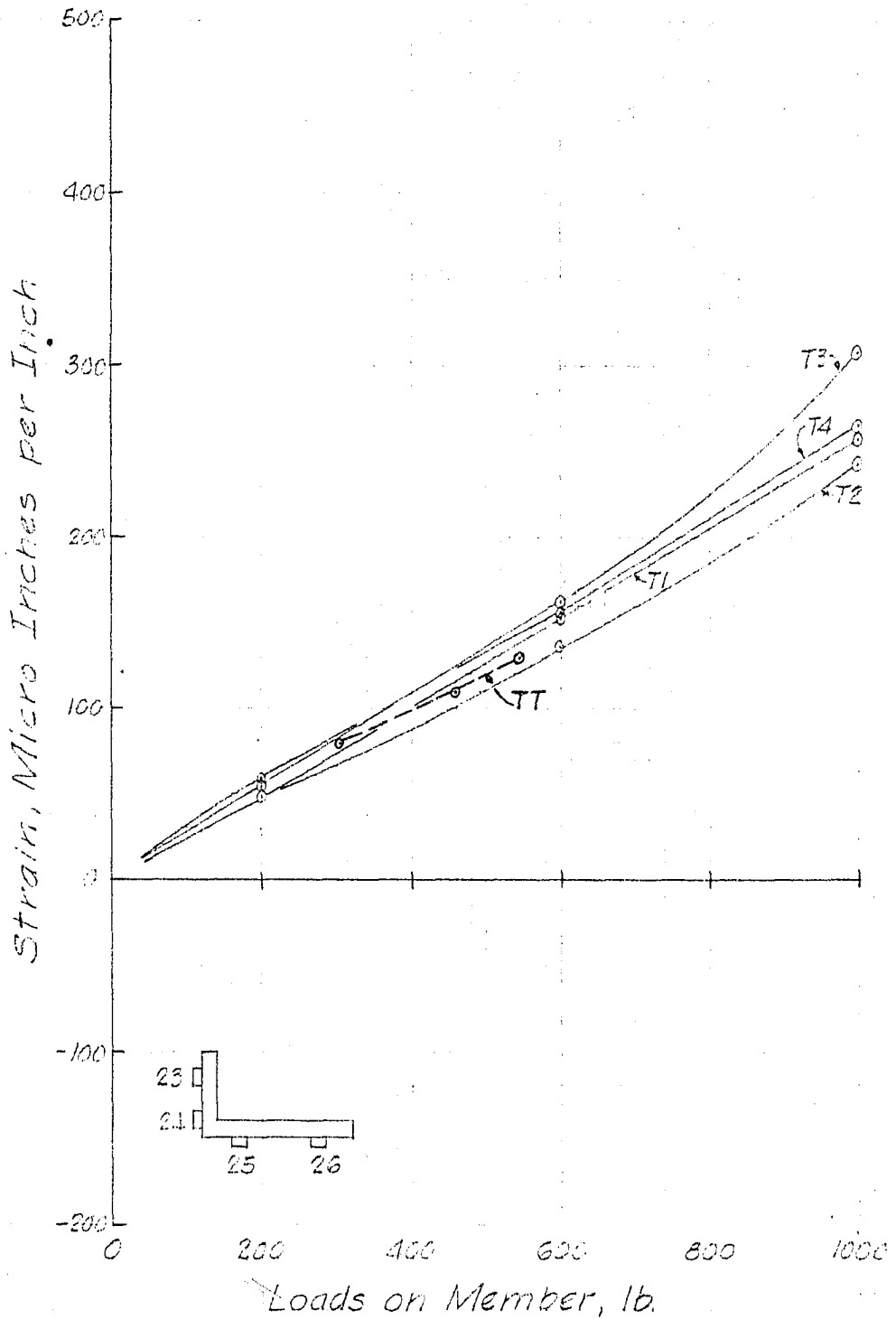
for Gage 23.

Figure A24. Member D7 in Tension, Load-Strain Curves

for Gage 24.

128
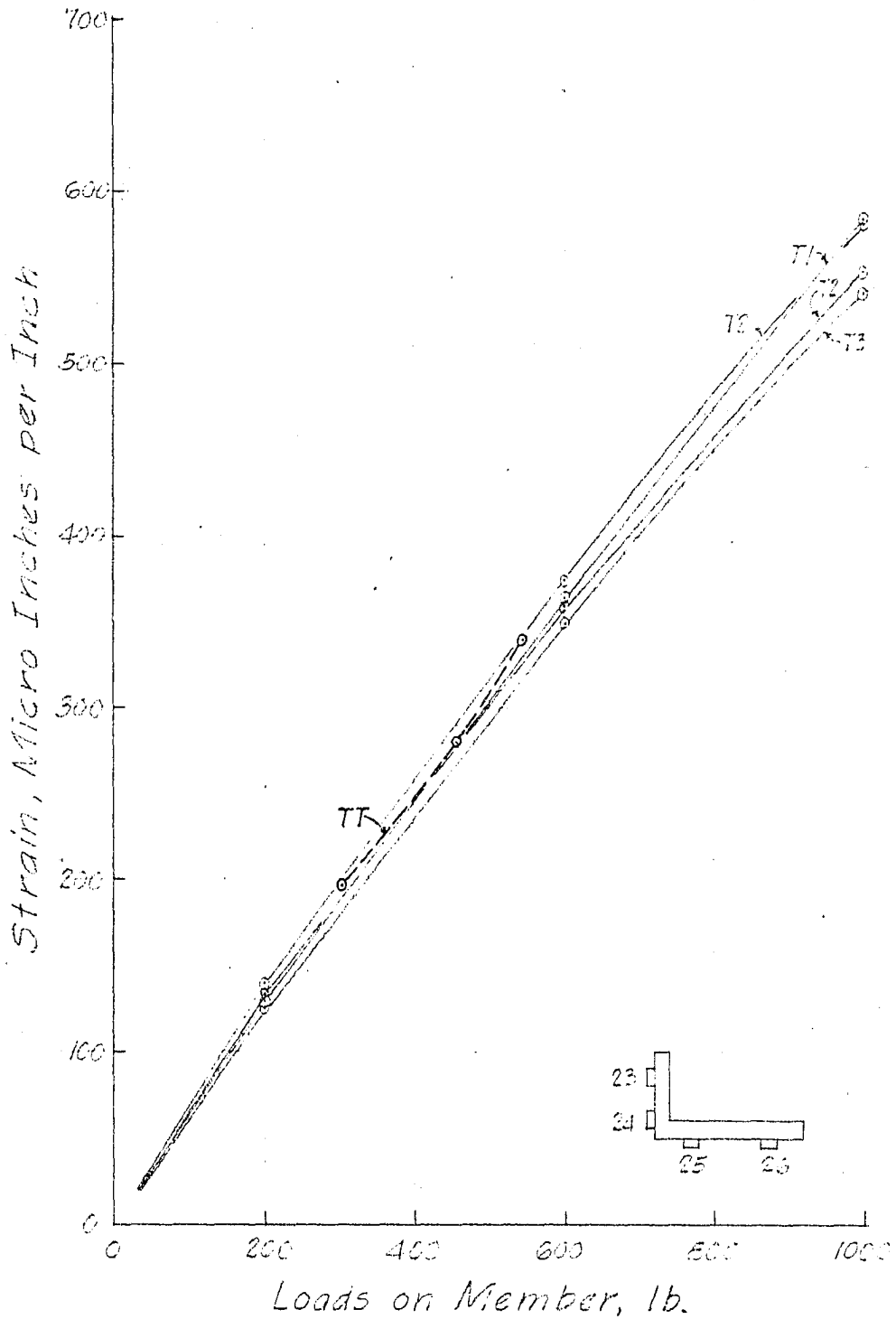
Figure A25. Member D7 in Tension, Load-Strain Curves
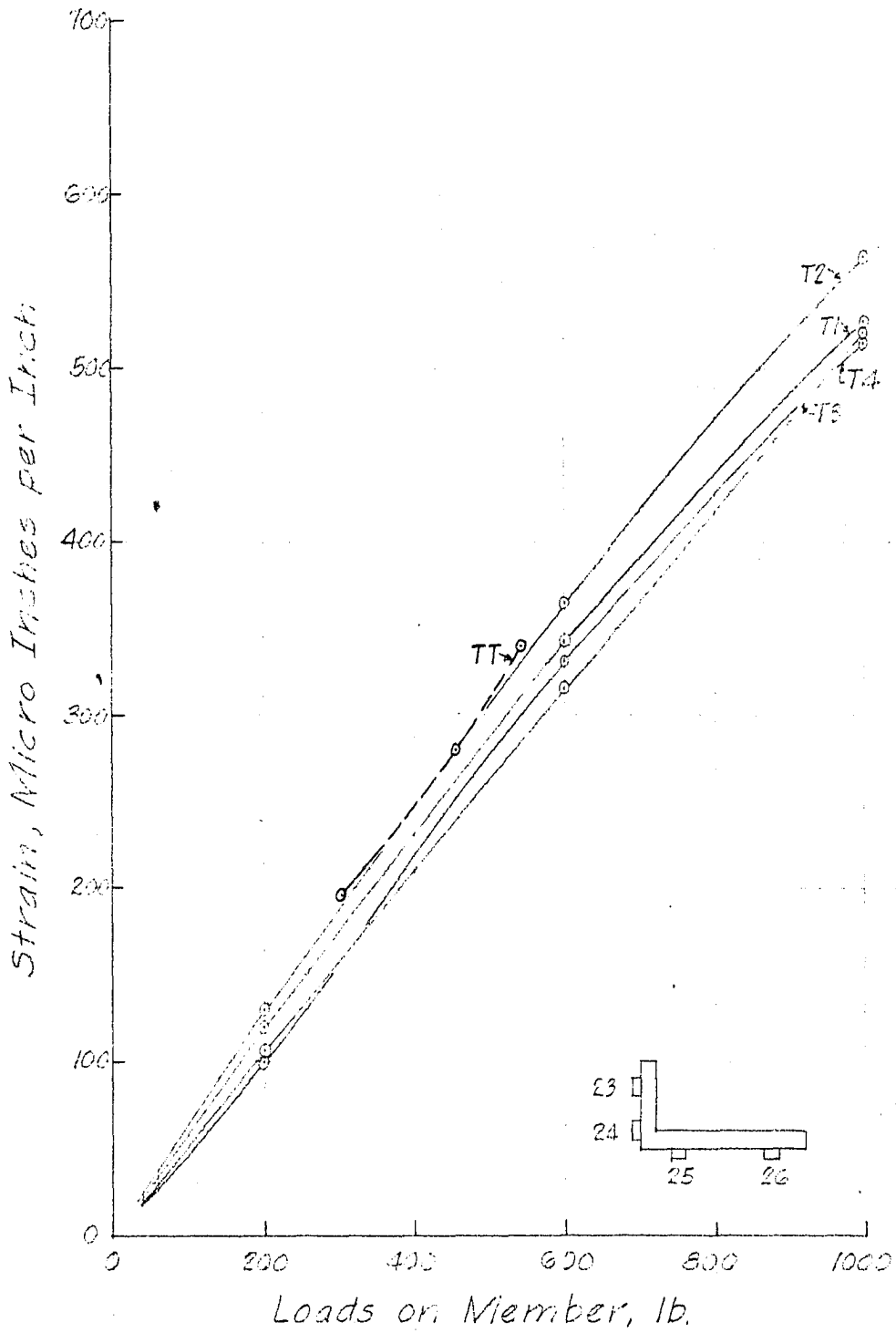
for Gage 25.

Figure A26.   Member D7 in Tension, Load-Strain Curves

for Gage 26.
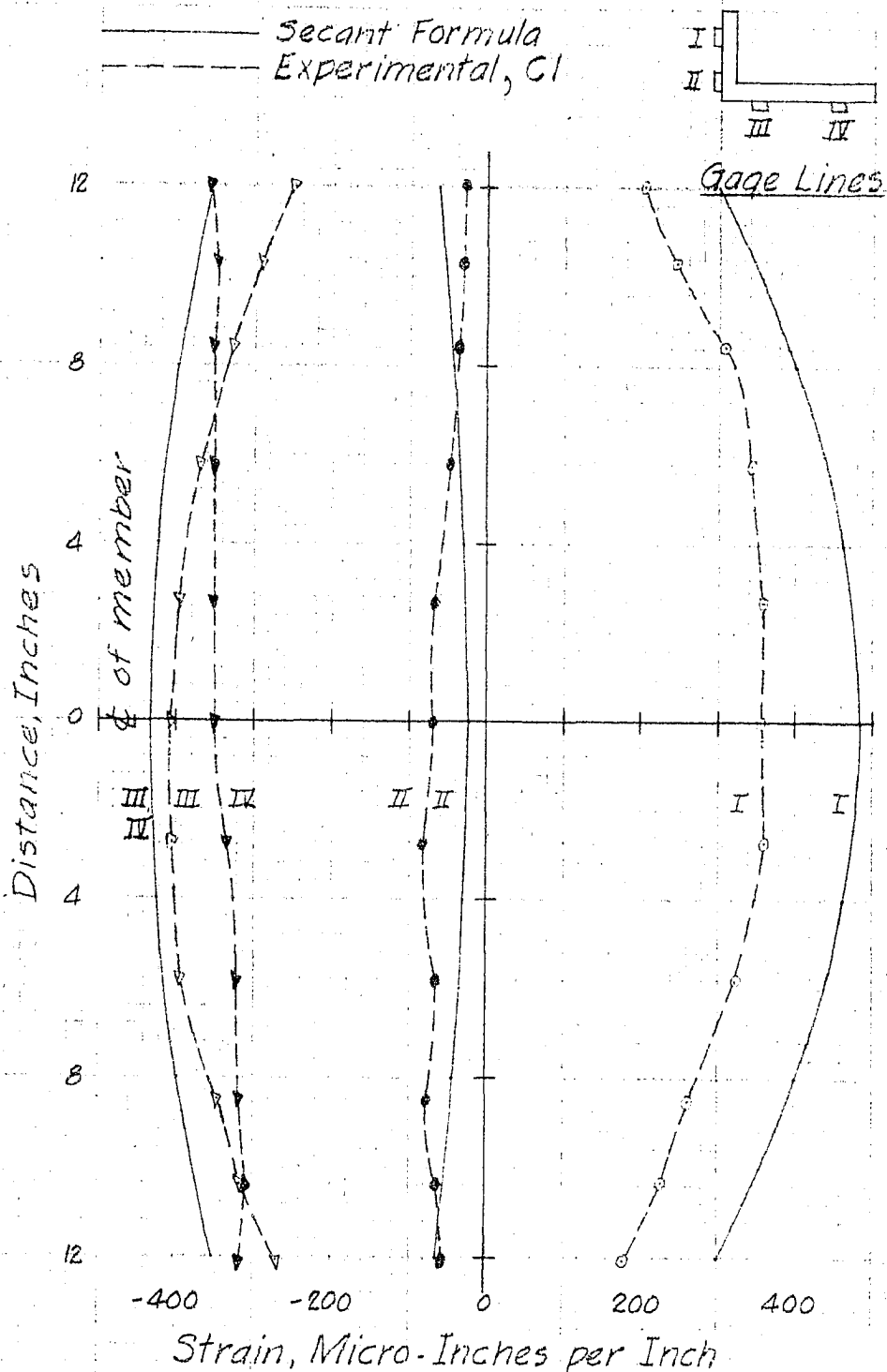
Figure A27. Member D5 in Compression, Comparison of Secant Formula and Condition C1 for Load of 449 lb.

131
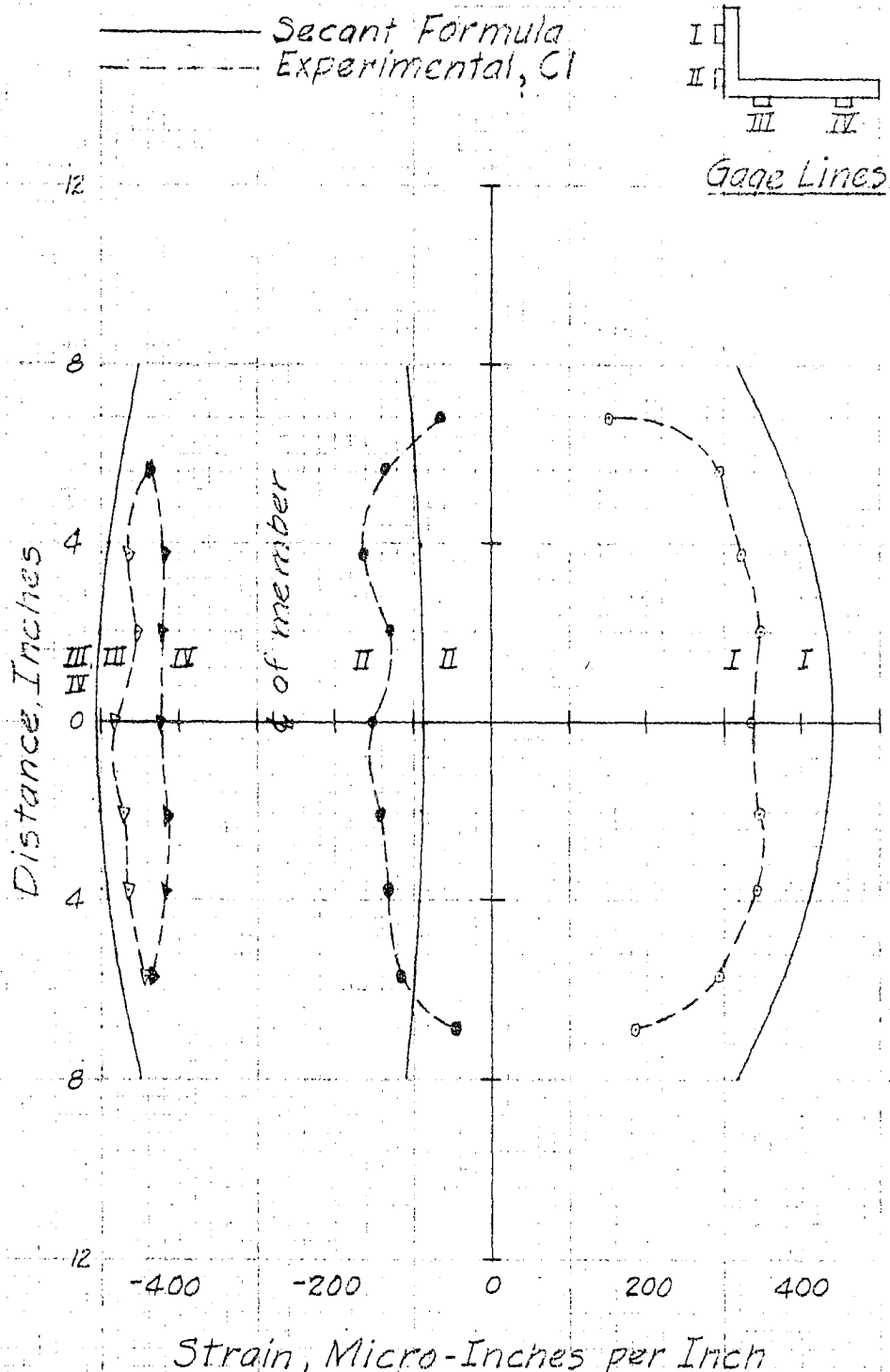
Figure A28. Member D7 in Compression, Comparison of Secant

Formula and Condition Cl for Load of 632 lb.
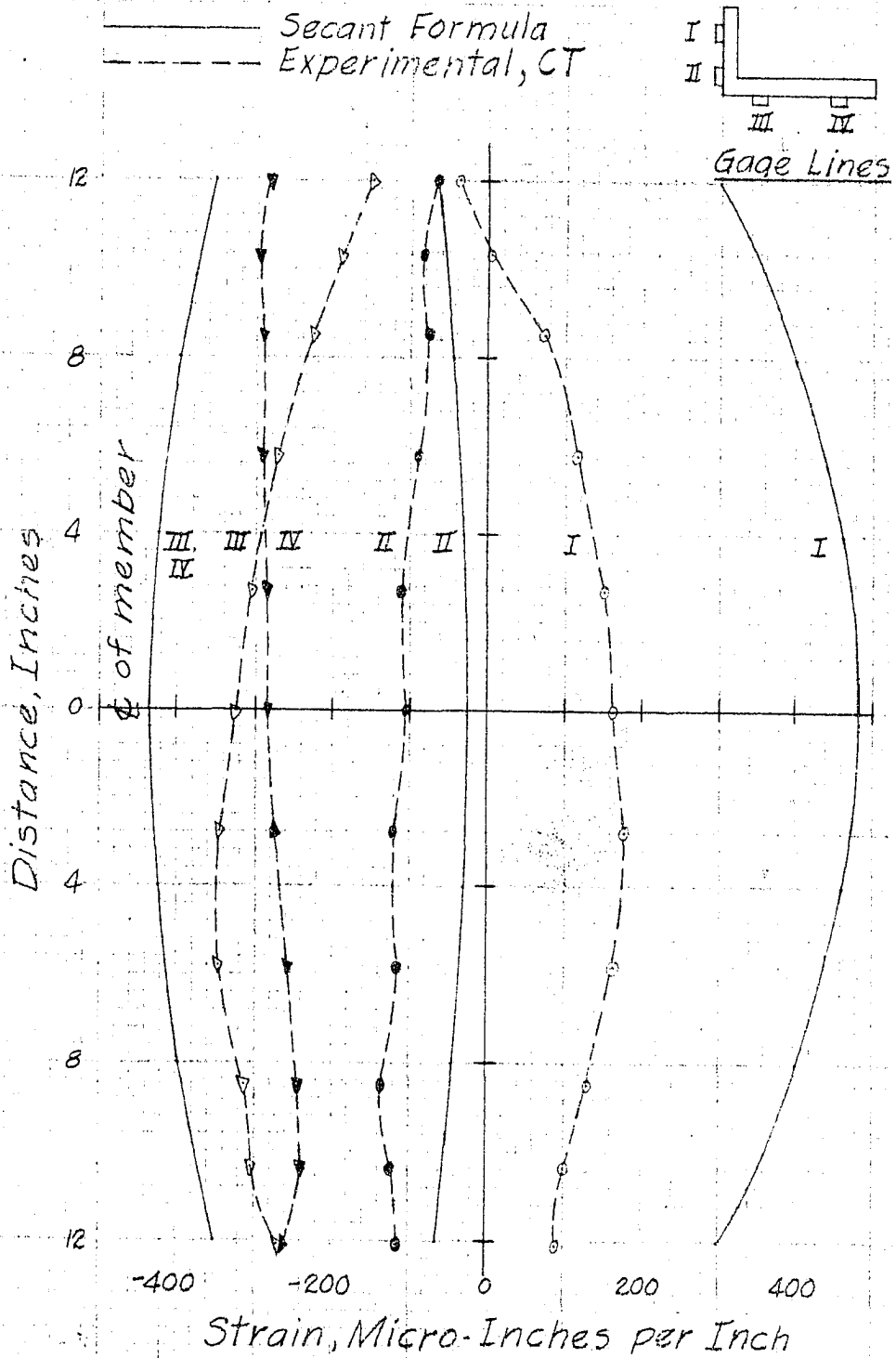
Figure A29. Member D5 in Compression, Comparison of Secant

Formula with Condition CT for Load of 449 lb.
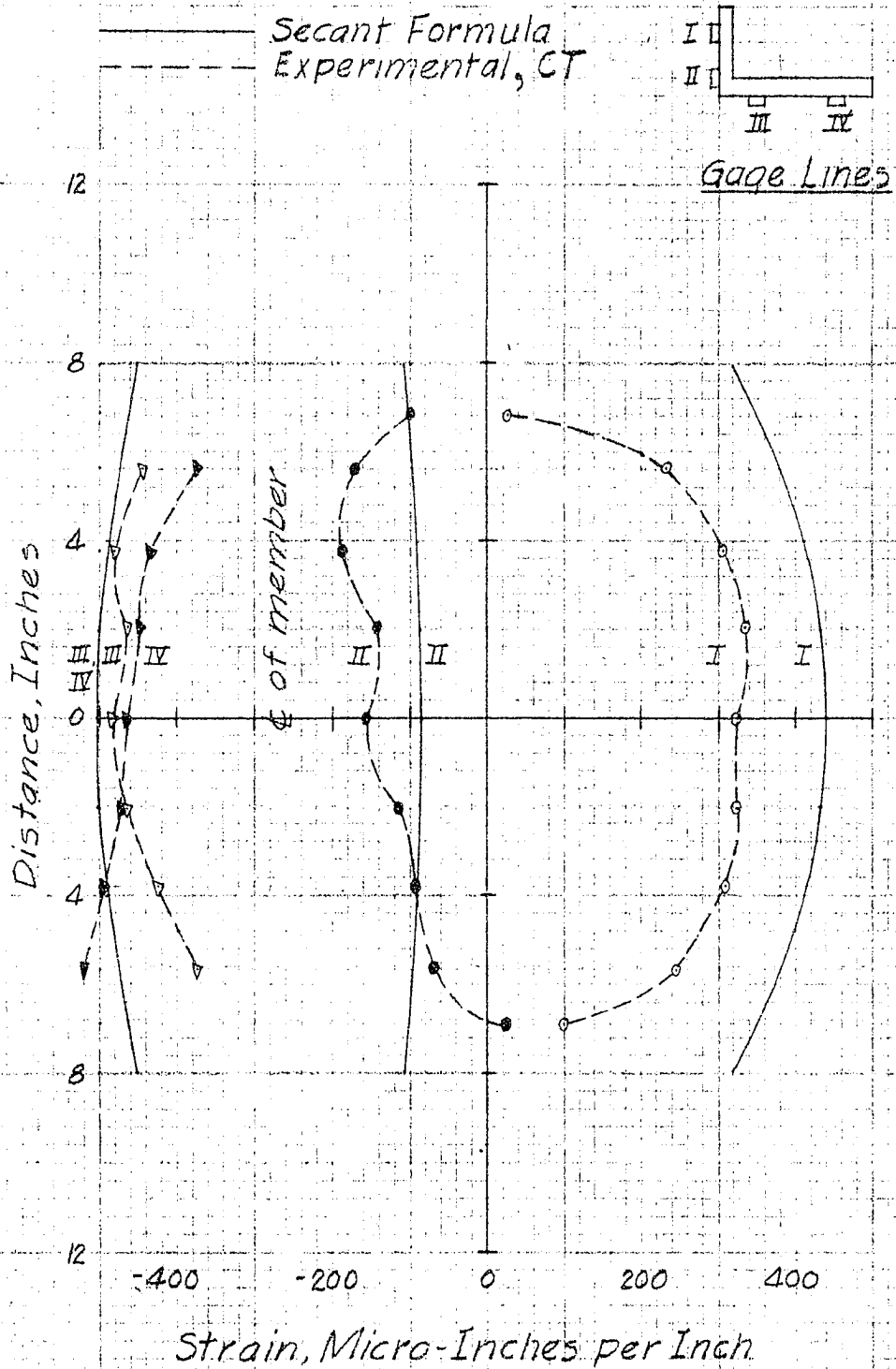
Figure A30.   Member D7 in Compression, Comparison of Secant
Formula and Condition CT for Load of 632 lb.

134
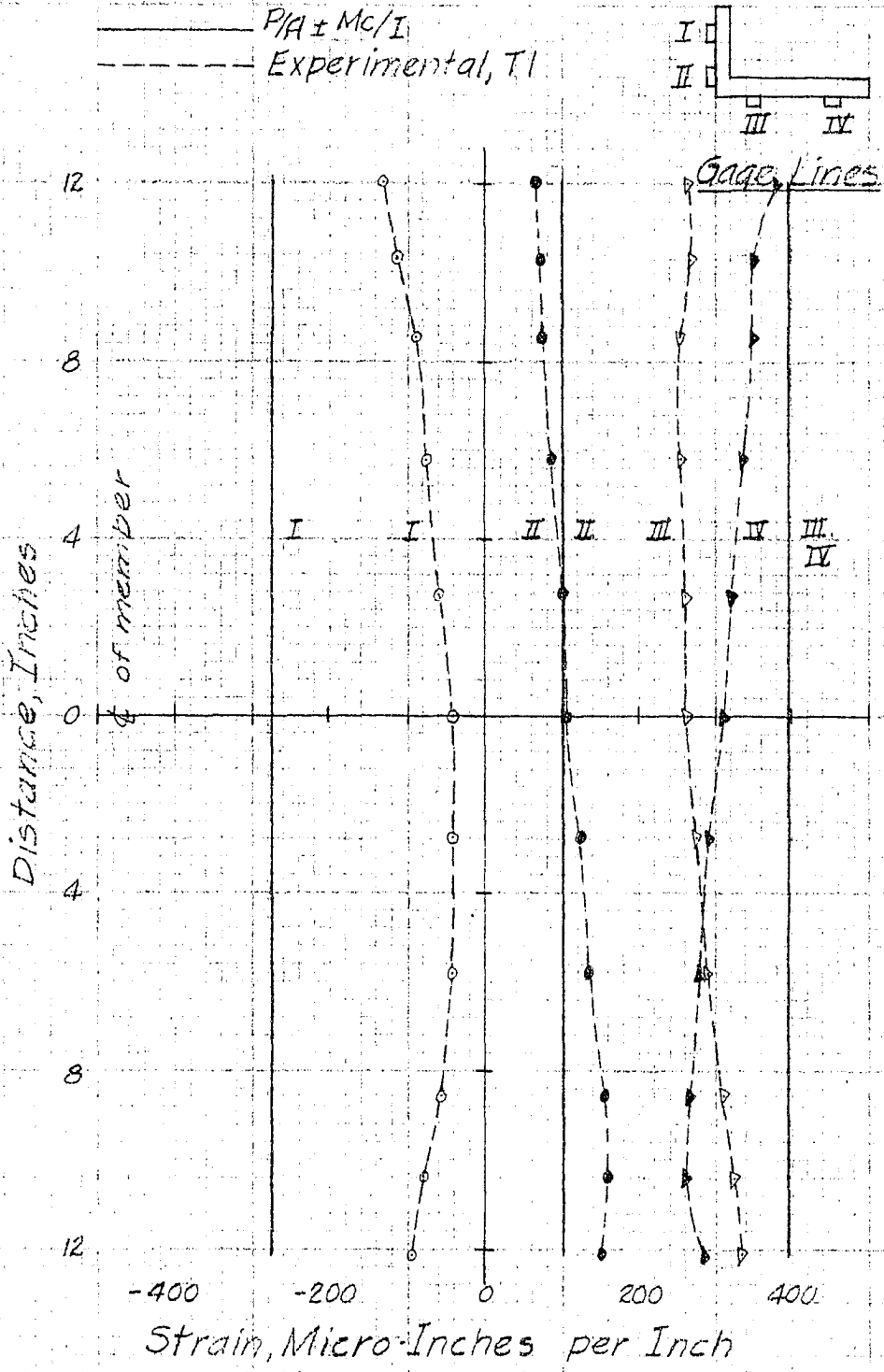
Figure A31.   Member D5 in Tension, Comparison of P/A ± Mc/I

and Condition T1 for Load of 550 lb.
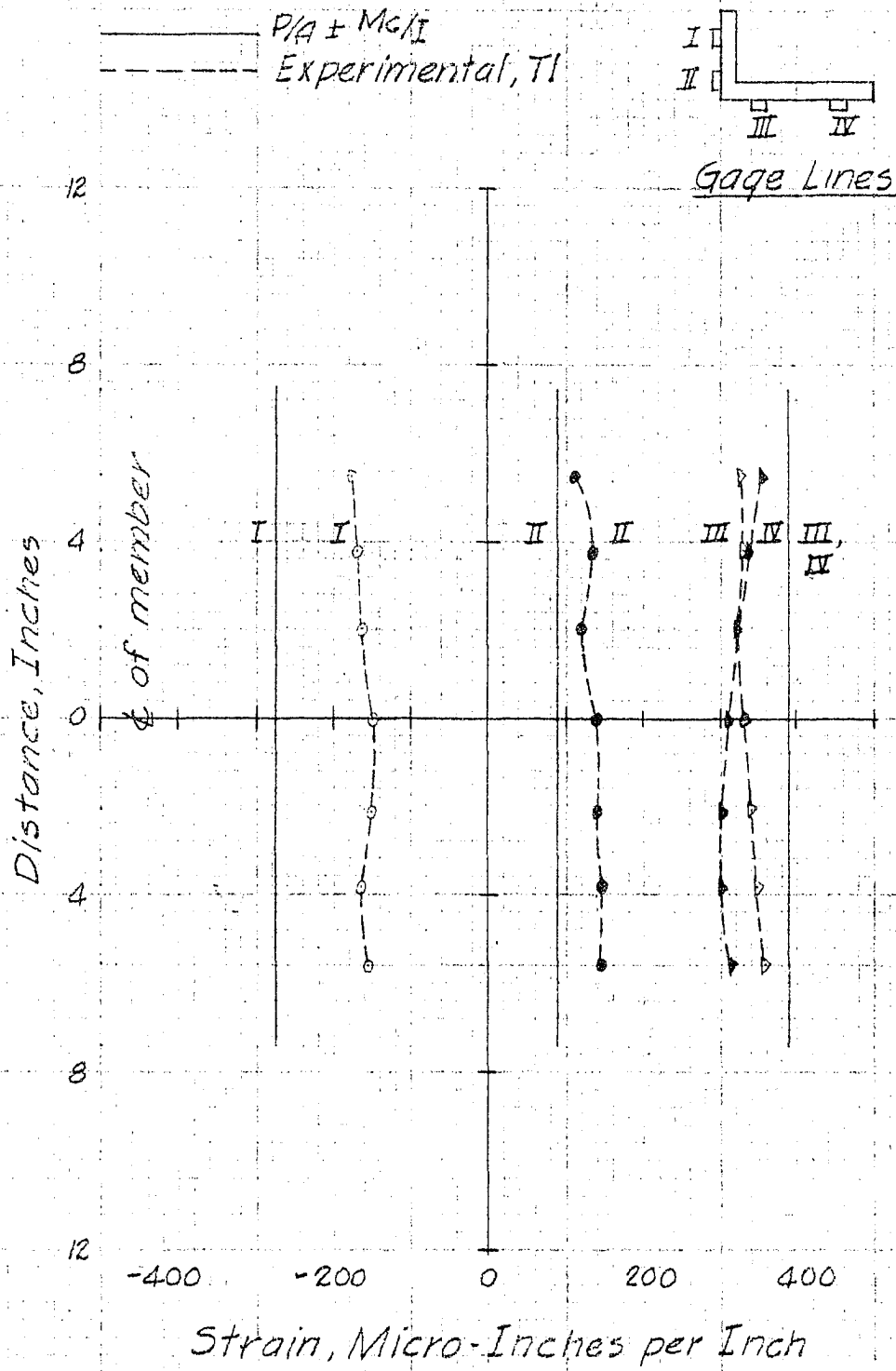
Figure A32. Member D7 in Tension, Comparison of P/A ± Mc/I
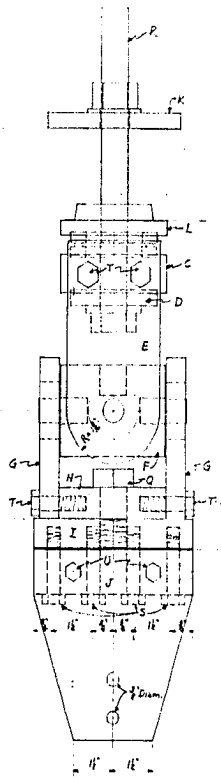and Condition T1 for Load of 544 lb.
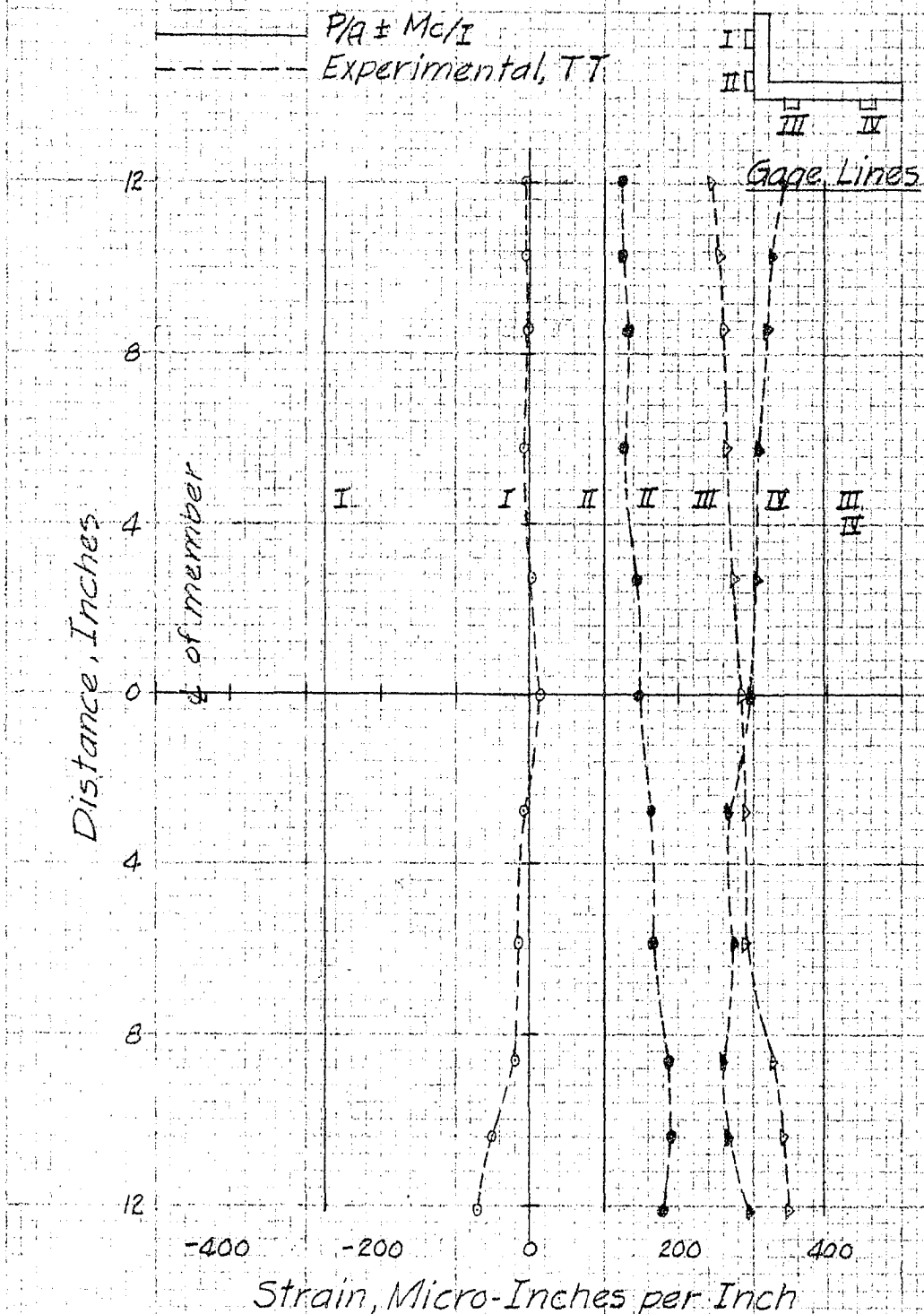
Figure A33. Member D5 in Tension, Comparison of P/A ± Mc/I and Condition TT for Load of 550 lb.
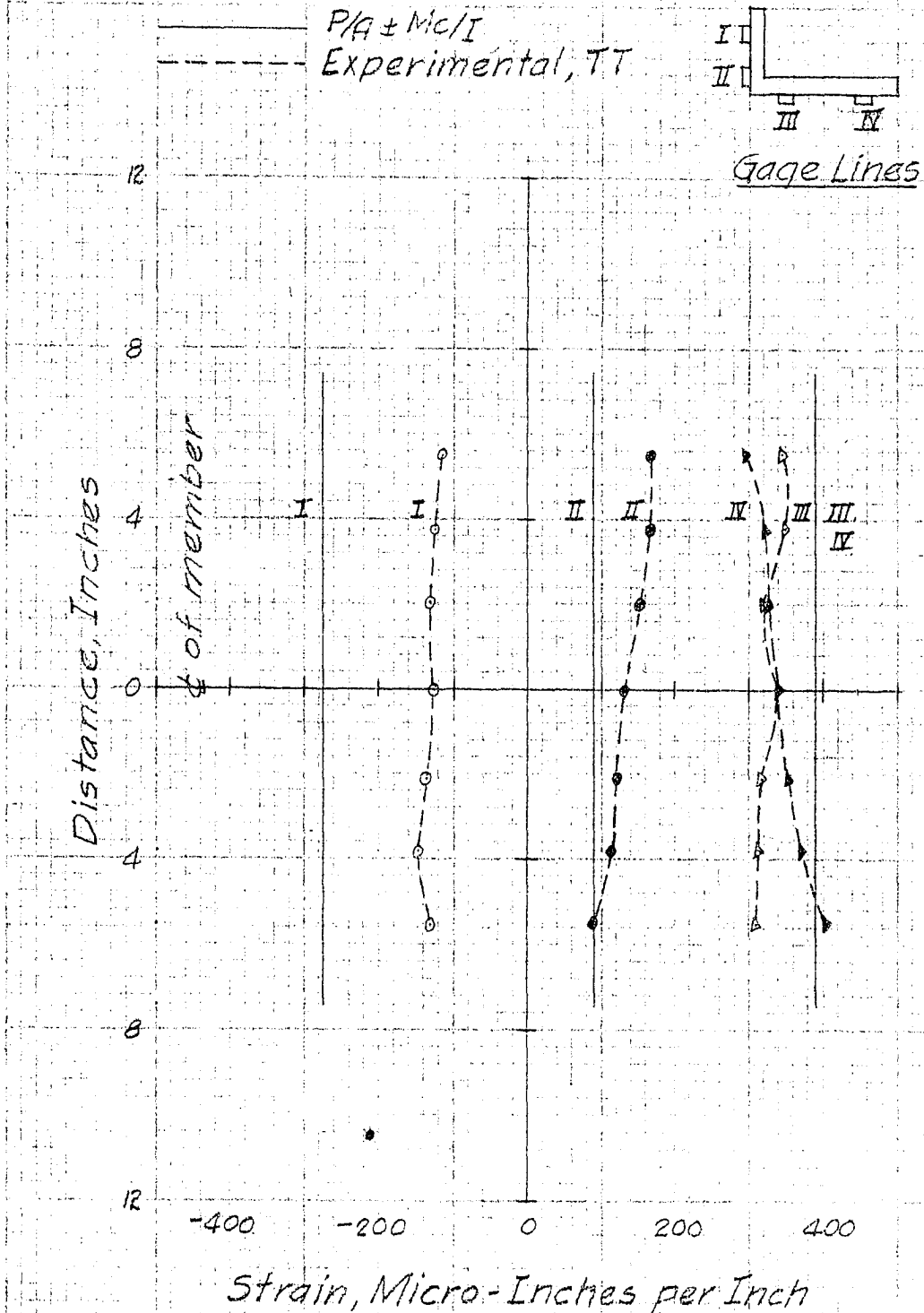
Figure A34. Member D7 in Tension, Comparison of P/A ± Mc/I
and Condition TT for Load of 544 lb.

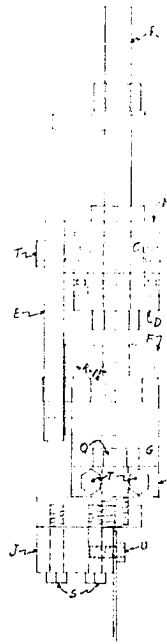B. Appendix B: Detail and Assembly Drawing of End Fixtures.

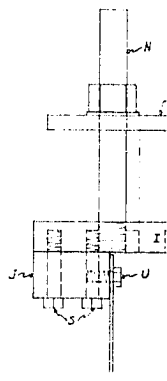Figure B1. Detail and Assembly

Drawing of End Fixtures.

ASSEMBLY FOR COMPRESSION
Scale: One Half Size

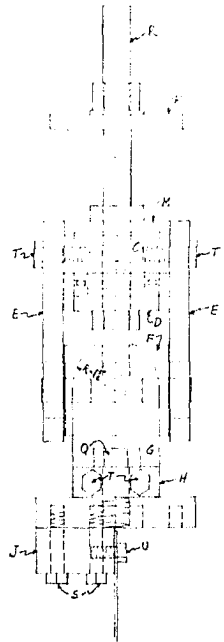| MATERIAL		
Mark	N°	Size
A	1	2".7".7"
B	1	⅝".4".4"
C	2	1¼".4".4"
D	2	4".3⅝"6
E	4	4".3⅝".78"
F	2	3".4".4"
G	4	4".3⅝".5⅝
H	2	1".3⅝".4"
I	2	1".6".6"
J	2	1⅝".3".6"
K	2	⅝".5".5"
L	1	1".4".4"
M	2	1".4".4"
N	2	1'6".8"
P	1	1'6".5" bolt
Q	3	1'6".2"
R	2	1'6".10"
S	16	⅝".2½"
T	16	⅝".1⅝"
U	4	⅝".1"

Head of testing machine

Aluminum Plates ⅛" thick

Bed of testing machine
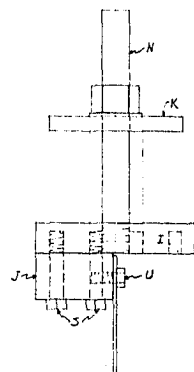
ASSEMBLY FOR Te
2 THUS

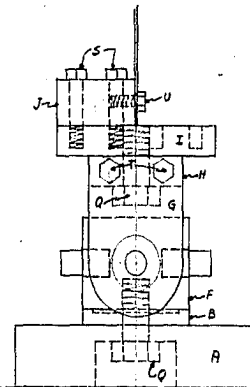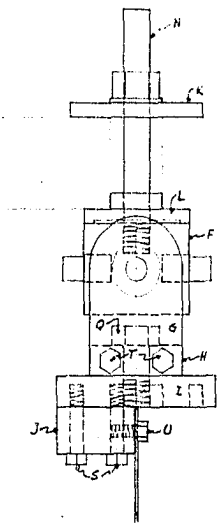ASSEMBLY FOR Te
OR COMPRESSI
2 THUS

ASSEMBLY FOR TENSION
2 THUS

ASSEMBLY FOR TENSION
OR COMPRESSION
2 THUS

ASSEMBLY FOR COMPRESSION
FOR TENSION 2 OF UPPER UNITS
ARE REQUIRED

END FIXTURES

MATERIAL	
Nº	Size
1	2".7".9"
1	2".4".4"
2	1½".4".4"
2	4". 3½"6
4	4". 3½.7⅛"
2	3". 4". 4"
4	4". 3½. 5⅝"
2	1". 3½". 4"
2	1". 6". 6"
2	1½". 3". 6"
2	5". 5". 5"
1	1". 4". 4"
2	1". 4". 4"
2	1"ø. 8"
1	1"ø. 5" bolt
3	1"ø. 2"
2	1"ø. 10"
16	½"ø. 2½"
16	½"ø. 1¾"
4	¾"ø. 1"

C. Appendix C. Determination of Modulus of Elasticity of
Material in Test Members.

A tension coupon was cut from the same piece of 14-SW
aluminum alloy angle from which all three of the test mem-
bers were fabricated. Two SR-4 gages, type A-12, were
attached on opposite faces of the tension coupon. The
coupon was loaded in tension in the testing machine by
increments of 200 lb. to a maximum load of 2800 lb. which
corresponded to a unit stress of 42,700 psi. Strain readings
were taken at each increment of load with the Scanning Re-
corder and the readings for the two gages were averaged at
each increment. The coupon was unloaded and the test was
repeated in the same manner. The results of the two tests
were then averaged as the final results. A stress-strain
curve was plotted and appears in Figure C1.

The modulus of elasticity of the material was determined
as

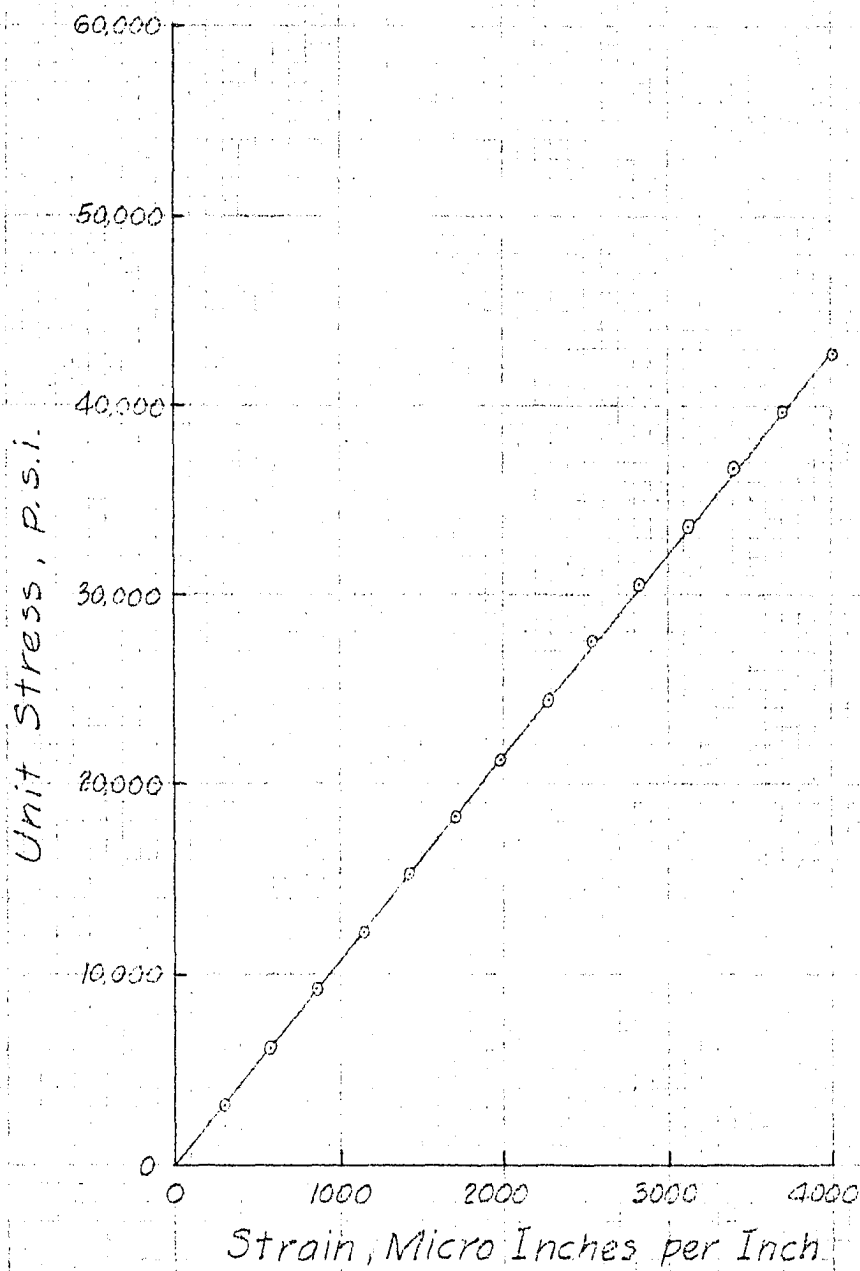$$E = \frac{32,100 \times 10^6}{3000} = 10,700,000 \text{ psi.}$$

Figure Cl. Stress-Strain Curve for Material in Test Members.